Journal of Physics Special Topics

An undergraduate physics journal

P5_1 Conditions for Lunar-stationary Satellites

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November 5, 2018

Abstract

This paper will explore what size and mass of a Moon-like body would have to be to have a Hill Sphere that would allow for lunar-stationary satellites to exist in a stable orbit. The radius of this body would have to be at least 2500 km, and the mass would have to be at least 2.1×10^{23} kg.

Introduction

Geostationary satellites are commonplace around the Earth. They are useful in providing line of sight over the entire planet, excluding the polar regions [1]. Due to the Earth's gravitational influence, lunar-stationary satellites are impossible, as the Moon's smaller size means it has a small Hill Sphere [2]. This paper will explore how big the Moon would have to be to allow for lunar-stationary satellites to orbit it.

Theory

Everything with a mass has a Hill Sphere, which is the area within which the body dominates the attraction of satellites [2]. Outside of this sphere of influence, the gravitational effect of other astronomical bodies will pull the satellite from orbit. In this case, satellites orbiting the Moon will be pulled into an orbit around Earth. Therefore stable orbits only exist within the Hill Sphere of an astronomical body.

Finding whether the Moon can host lunar-stationary satellites first requires the semi-major axis at which a satellite must orbit at to be concidered lunar-stationary. The moon has a rotational period of around 27 days [3], which means any lunar-stationary satellite must also have an orbital period of 27 days. We can calculate the orbital radius of the satellite using the following equation [3]:

$$T = 2\pi\sqrt{r^3/GM_m}, \tag{1}$$

In equation 1, r is the orbital radius, G is the gravitational constant, M_M is the mass of the Moon, and T is the orbital period. Solving for r gives a radius of around 88000 km. When we compare the lunar-stationary radius to the Moon's Hill Sphere of around 60000 km [2], we can see that lunar-stationary satellites are not possible. For lunar-stationary satellites to remain in a stable orbit, the Hill Sphere of the Moon must therefore extend to 88000 km. The equation for the Hill Sphere of a body is as follows [4]:

$$r_H = a\sqrt[3]{M_m/3M_e}, \tag{2}$$

For equation 2, a is the semi-major axis of the Moon's orbit around the Earth, M_M is the mass of the Moon, and M_E is the mass of the Earth. From this, we can calculate the required mass to have a Hill Sphere of 88000 km, which was calculated as 2.1×10^{23} kg. Then, using the density of the Moon, which is 3.3 g/m^3 [5], we can then find the radius of this theorised body that would allow a satellite to have an orbital period which is the same as one rotation of the body. We have to assume with this that the Moon is uniformly dense and spherical. For this we use the following equation [3]:

$$\rho = 3M/4\pi r^3, \tag{3}$$

Where ρ is the density of the body, M is its mass, and r is its radius. This gives a new lunar radius of around 2500 km.

Discussion

As calculated in the theory, for satellites to be in a stable lunar-stationary orbit, the Moon must grow by around four times its mass and around one and a half times its radius. At this size, the Moon would have a larger effect on the Earth, such as much more dramatic tidal changes. Additionally, the Moon is responsible for the slowing down of the Earth's rotation due to tidal friction by a fraction of a second every year [6]. A more massive Moon would increase the rate of this slowing, and hence the length of a day on Earth would be much longer.

Conclusion

In order to increase the Moon's Hill Sphere to a distance that would allow geostationary satellites to orbit, its mass would have to increase to 2.1×10^{23} kg, around four times its current mass. Assuming the density of the Moon would remain constant, the lunar radius would also increase to around 2500 km.

References

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