Journal of Physics Special Topics
An undergraduate physics journal

A4_6 Banana Power

R. Pearce, J. Buckland, O. James, M. Crosby

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 17, 2017

Abstract

In this paper we investigated the feasibility of using the radioactive potassium in bananas, as a fuel source for a radioisotope thermo-electric generator (RTG). We have calculated that 1.1×10^{17} bananas are needed to power a 100 W light bulb, resulting in a spherical fuel cell of radius 15×10^{3} m, and weighing 1.3×10^{16} kg, from which we determined bananas are impractical to fuel an RTG.

Introduction

It is well known that bananas contain noticeable levels of potassium, to the extent the Banana Equivalent Dose - a measure of the standard radiation dose recieved from eating one banana - is used in general media [1]. By harnessing the energy released by the decay of radioactive Potassium-40 in the banana, which accounts for 0.012% of naturally occuring potassium [2], theoretically we could fuel a radioisotope thermoelectric generator (RTG) with a small power output. The average power output of a banana can be calculated using: the decay rate of Potassium-40, the percentage of decays that produce harnessable energy, a banana's potassium content, and mass of an average banana. It is then trivial to calculate the number of bananas, and size of generator needed (spherical to minimise surface area and so energy loss to the surroundings) to power a standard 100 W light bulb.

A Banana's Energy

We can calculate the decay rate, R, by substituting Eq. (1) into Eq. (2), to give Eq. (3),

$$R_0 = \lambda N_0, \tag{1}$$

$$R = R_0 \exp(-t\lambda), \tag{2}$$

$$R = \lambda N_0 \exp(-t\lambda). \tag{3}$$

R_0 is initial decay rate, λ is the decay constant (equal to the inverse of the half-life), t is the time, and N_0 is initial number of undecayed nuclei [3].

Potassium-40 can decay in three ways; 89% by β^- decay, 11% by electron capture emitting a gamma ray of 1.46 MeV, and 0.001% by β^+ decay [4]. The β^- decays are ignored as its products are not harnessed by the RTG. Assuming for β^+ decay, the positron will quickly annihilate within the mass of the bananas, it will release two photons of 511 keV each in opposite directions [5]. Using the energy released through positron annihilation and gamma ray emission, we can find how many bananas are needed to light a standard 100 W light bulb using a RTG.

For a medium sized banana ($m_B = 118$ g [6]) the potassium content is 0.422 g [6], of which only 5.0×10^{-5} g will be Potassium-40. Therefore each banana will contain 1.3×10^{-6} mol, or 7.2×10^{17} nuclides, of Potassium-40.

The initial λ, at time $t = 1$ s, is found using,

$$\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}, \qquad (4)$$

where the half-life of Potassium 40 is 1.251 billion years [7]. Using Eq (4), λ can be calculated to be 7.99×10^{-10} s^{-1}. For a single banana, the decay rate for only gamma-emitting decays would be 0.066 s^{-1} producing 0.097 MeV s^{-1}, and for positron-emitting decays it would be 6×10^{-6} s^{-1} producing a comparatively small 3 eV s^{-1}. Combined, this amounts to a power output of $P_B = 0.097$ MeV s^{-1} per banana.

Radioisotope Thermo-Electric Generator

RTGs are used to convert heat produced by radioactive decay into electricity through the use of thermocouples - currently the efficiency of such generators is about 6% [8]. However this is the efficiency for RTGs used in space, where the outside environment would be much colder than on Earth, so the efficiency would likely be higher. By using the power produced by one banana (P_B), we can estimate the number needed (N_B) to fuel a RTG, giving a total power (P_T) of 100 W for a light bulb.

$$N_B = \frac{P_T}{P_B \times efficiency} \qquad (5)$$

Using Eq. (5) we calculate the total number of bananas required would be $N_B = 1.1 \times 10^{17}$ [9]. The volume (V) of the banana fuel cell is

$$V = \frac{m_{Btotal}}{\rho_B} = \frac{4}{3}\pi r^3, \qquad (6)$$

where m_{Btotal} is the total mass of the banana, the density of banana $\rho_B = 951.02$ kg m^3, and r is the radius of the spherical fuel cell.

Rearranging Eq. (6) for r and by substituting in m_{Btotal} for $m_B \times N_B$ gives

$$r = \frac{3m_B N_B}{4\pi\rho_B}^{\frac{1}{3}}. \qquad (7)$$

Using Eq. (7), the generator can be found to need a ball of banana with a radius of 1.5×10^4 m and weighing 1.3×10^{16} kg.

Conclusion

We have found that to generate 100 W using a banana powered RTG, 1.1×10^{17} bananas weighing 1.3×10^{16} kg would be required. This is one hundred thousand times the mass of bananas grown in 2013 [10]. The spherical fuel cell would have a radius of 1.5×10^4 m which is prohibitive for any practical use, especially when considering the small amount of power it produces.

References

[1] http://www.bbc.co.uk/news/magazine-1528895 [Accessed 29 November 2017]

[2] http://phi.nmsu.edu/~pvs/teaching/phys593/potassium.pdf [Accessed 12 November 2017]

[3] P. A. Tipler, G. Mosca, *Physics for Scientists and Engineers*, (New York, W. H. Freeman and Company, 2008), 6th edition.

[4] D. W. Engelkemeir, K. F. Flynn, and L. E. Glendenin, *Phys. Rev. 126*, p. 1818-1822, 1962

[5] https://radiopaedia.org/articles/electron-positron-annihilation [Accessed 12 November 2017]

[6] https://ndb.nal.usda.gov/ndb/search/list/ [Accessed 12 November 2017]

[7] http://www.radioactivity.eu.com/site/pages/Potassium_40.htm [Accessed 12 Novemeber 2017]

[8] http://www.gao.gov/assets/690/687031.pdf [Accessed 14 November 2017]

[9] https://www.aqua-calc.com/page/density-table/substance/bananas-coma-and-blank-raw-coma-and-blank-skin-blank--op-cup-coma-and-blank-mashed-cp- [Accessed 14 Novemeber 2017]

[10] http://www.bananalink.org.uk/all-about-bananas[Accessed 14 Novemeber 2017]