

S3_4 We're Walking in the Air

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Abstract

In the popular 1982 TV adaptation of *The Snowman*, a young boy wakes up in the morning to find his snowman has melted overnight. We investigated whether this would be realistic. We have calculated how long it would take for a 1.8 m tall snowman to melt based on the power from the Sun. With complete absorption of light, the snowman would take 16.3 hours to melt. This number increases when considering incomplete absorption of light.

Introduction

We explore the melting rate of snow assuming all heat comes from the Sun's rays, and that the Sun is directly overhead of the snowman. We neglect the ambient temperature of the surroundings as these are assumed to be low, and effects from the humidity (latent heat of vaporisation) tend to cancel out the effects of the ambient temperature [1]. We also assume that the snowman is made of three spheres of compressed snow of diameter 40cm, 60cm and 80cm respectively.

Theory and Results

We modelled the snowman as 3 spheres of ice stacked on top of each other, as shown in Figure 1(a). The spheres are of diameter $d_1=40$ cm, $d_2=60$ cm and $d_3=80$ cm. To calculate the volume of each sphere we used $V = \frac{4}{3}\pi r^3$ to find $V_1=0.0335$ m³, $V_2=0.113$ m³ and $V_3=0.286$ m³. We then calculated the total mass of the snowman by using Equation 1

$$m_{tot} = \rho_{snow} \sum_n V_n \quad (1)$$

using ρ_{snow} as 200 kgm⁻³ [2], and where V_n is

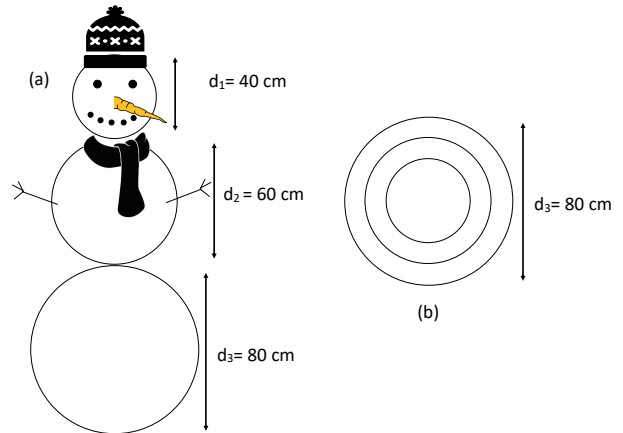


Figure 1: A diagram showing (a) the proportions of the snowman, (b) an overhead view of the snowman

the volume of each snow sphere where $n=1,2,\dots,n$ to find a total mass, $m_{tot}=82.9$ kg. We then calculated the heat required to melt the snowman using Equations 2 and 3, using L_f as 333.5 kJkg⁻¹ [2], c as 2.05 kJkg⁻¹K⁻¹ [2] and the change in temperature of -10 C to 0 C.

$$Q_f = m_{tot}L_f \quad (2)$$

$$Q_T = m_{tot}c\Delta T \quad (3)$$

Where Q_f is the the heat required to change phase, m_{tot} is the mass of the snowman, L_f is the latent heat of fusion of compressed snow, Q_T is the heat required to change the temperature, c is the specific heat capacity and ΔT is the change in temperature.

We summed these energies to gain a total energy Q_{tot} of 29 MJ. This is the total energy required to melt the snowman. As previously stated, this energy comes from the intensity of the Sun. The intensity of the Sun at the top of the Earth's atmosphere is 1360 Wm^{-2} [3], so we took the intensity of the Sun reaching the Earth's surface as approximately 1000 Wm^{-2} , to take into account the energy lost to the atmosphere. Assuming the sunlight hits the snowman directly overhead, as shown in Figure 1(b), we calculated the area of the snowman exposed to the Sun to be 0.50 m^2 . Multiplying this area by the intensity from the Sun, we found that the snowman would be subject to 500 W of power from the Sun. To find the time taken for the snowman to melt we used Equation 4.

$$t = \frac{Q_{tot}}{P} \quad (4)$$

where t is time in seconds, and P is the power. From this we found that the snowman would take 59000 seconds, or approximately 16 hours to melt.

This solution does not take into account the fact that snow does not absorb all radiation incident on it. In reality, snow reflects a large amount of the sunlight incident on it, and this varies dependent on how compact the snow is and impurities in the snow. Figure 2 is a plot of the time taken for the snowman to melt versus the percentage of light that the snow absorbs, showing that the snowman could take over 150 hours to melt.

Conclusion

We investigated the time taken for the snowman from the television film *The Snowman* to melt from -10 C to 0 C based on the intensity of the Sun's rays. We found that if the snowman absorbed all of the sunlight it would take

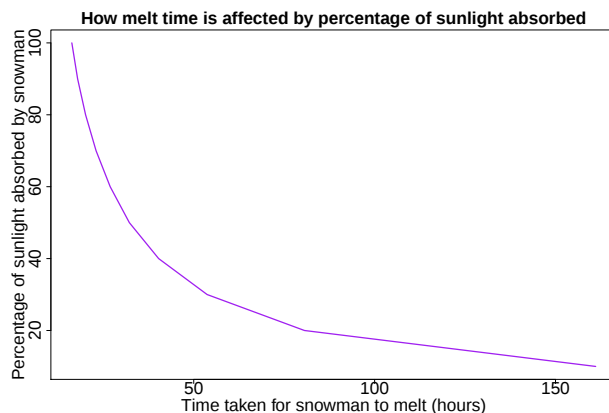


Figure 2: A graph of percentage of incident sunlight absorbed by the snowman against time taken for the snowman to melt

16 hours to melt, which could be reasonable, as the boy would probably have slept for around 8-10 hours, as well as spending the previous day with the snowman, during which time the melting process would have started. However, when taking into account Figure 2, assuming the snow only absorbs 50% of the light incident on it, the snowman would actually take much longer, approximately 30 hours. It is worth noting also that during the 8-10 hours when the boy was sleeping, the snowman would not have undertaken much melting as it would have been dark, therefore no sunlight would be incident on the snowman in this time. Taking all of this into consideration, the boy in the film would not have woken up to his snowman having melted, and the story need not have a sad ending.

References

- [1] Evans, H. et al <https://directives.sc.egov.usda.gov/OpenNonWebContent.aspx?content=17753.wba> (2004) [Accessed 15 Nov. 2017].
- [2] P. Tipler, G. Mosca, Physics for Scientists and Engineers, (W.H. Freeman and Company, New York, 2008), 6th edition.
- [3] <https://earthobservatory.nasa.gov/Features/EnergyBalance/page2.php> (2017) [Accessed 15 Nov. 2017]