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A3_2 It's Snow Problem

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Abstract

Falling into a pile of snow can sometimes feel like falling into a cold, wet and slightly uncomfortable cushion. Although it may not be the most enjoyable way to break a fall, the fact remains that snow can dampen the force you experience and subsequently make the fall less painful. In this article we calculate how much snow it would take to make a fall at terminal velocity, survivable. For a 70kg body falling at terminal velocity, the height of snow required to reduce the deceleration to a survivable 35 g was found to be 5.08 m.

Introduction

The injuries you may sustain from falling over are not from the act of falling itself. Instead, they are the product of instantaneously coming to a stop the moment you make contact with the floor. One way of decreasing this stopping force is to increase the duration of the deceleration period to one long enough to generate a g force that can be endured by the human body. We propose the idea of using snow as a buffer to absorb the impact of such a fall. Using the compression of the snow into ice as a deceleration period, this will give us the average deceleration experienced by the person.

Terminal Velocity

To obtain a value for the velocity with which a body would hit the snow, the weight and drag forces experienced by said body are equated and rearranged for v_t. This gives the following equation [1],

$$v_t = \sqrt{\frac{2mg}{C\rho A}}, \tag{1}$$

where v_t is the terminal velocity, m is the mass of the body, g is the acceleration due to gravity (assumed to be constant over the distance of the fall), C is the drag coefficient for the body, ρ is the density of air and A is the projected area in the direction of travel. Substituting in a mass of 70 kg, an area of 0.7 m^2 [2], a drag coefficient of 1.2 [3] and an air density of 1.2 kgm^{-3} [4] gives a terminal velocity of 36.9 ms^{-1}. This value is used for the initial velocity when performing the deceleration calculation.

Depth of Snow

When snow is compressed by a force, exerted by a falling body in this case, it becomes increasingly more compact and eventually turns into ice. The point at which the snow becomes ice is the point at which the body stops its fall. The distance over which the deceleration of the body occurs is the difference between the initial depth of uncompressed snow and the final depth of the newly compacted ice. In this article we assume that the area of the snow compacted will be the same, so that only depth varies in the considered volume of snow. Also assumed is that the initial mass of snow will be compressed until it becomes ice of the same mass. Additionally we assume that the body's deceleration will be constant throughout the impact, providing us with a 'best case scenario'; in reality the deceleration would presumably be exponential due to the snow increasing in density throughout the collisional compression. Taking the density of settled snow and ice to be 200 kgm^{-3} and 917 kgm^{-3} [5] respectively, the ratio between the depth of the snow and depth of an equal mass of ice was found using,

$$M_S = \rho_S V_S = \rho_S A_S D_S = M_I \tag{2}$$

$$M_I = \rho_I V_I = \rho_I A_S D_I \therefore D_I = \frac{D_S \rho_S}{\rho_I}. \tag{3}$$

Where D is depth, ρ is density, M is mass, V is volume, A is area and the subscripts S and I stand for snow and ice respectively.

Deceleration

To calculate the average deceleration experienced by the body, the equation of average velocity is used

$$t_s = \frac{\Delta D}{v_t} = \frac{D_S(\rho_I - \rho_S)}{\rho_I v_t}. \tag{4}$$

Where v_t is the terminal velocity, ΔD is the stopping displacement and t_s is the stopping time. Following from this,

$$a = \frac{v_t}{t_s} = \frac{v_t^2 \rho_I}{D_S(\rho_I - \rho_S)}, \tag{5}$$

where a is the average deceleration of the human. The largest g force that a human has survived is 46.2 g [6], a feat achieved by John Stapp using a rocket powdered sled propelled by 9 solid fuel rockets. He was accelerated to a velocity of 1017 kmh^{-1} in 5 s. We assume that if an acceleration of this scale is survivable, so is a deceleration of equal magnitude. However, this case is extreme, so we model our scenario using a slightly lower value for deceleration of 35 g; a value the average human is more likely to survive, sustaining only minor injuries. Thus, rearranging Eq. 5 for D_S and using this value of g, the required height of the snow was found to be 5.08 m.

Conclusion

It has be shown that settled snow of a depth of 5 m can slow even a deadly fall at terminal velocity to one that can be survived. The main limitation to this model materialises when not considering the rate of change of deceleration, a variable we assume to be zero; an improvement would be to take this into account to obtain a more accurate value for D_S.

References

[1] Tipler, P. and Mosca, G. (2008). Physics for scientists and engineers. New York: W. H. Freeman. Sixth Edition

[2] McIlveen, J (2002), The everyday effects of wind drag on people. `doi:10.1256/wea.29.02` [Accessed 20 October 2017]

[3] Koo et al, (2013) `http://www.taylors.edu.my/EURECA/2014/downloads/02.pdf` [Accessed 20 October 2017]

[4] `https://www.engineeringtoolbox.com/air-density-specific-weight-d_600.html` [Accessed 20 October 2017]

[5] `https://www.sciencelearn.org.nz/resources/1391-snow-and-ice-density` [Accessed 20 October 2017]

[6] Tyson, (2007) `http://www.pbs.org/wgbh/nova/space/gravity-forces.html` [Accessed 20 October 2017]