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S1_3 Eye See You Frodo

A. Gajendran, S. Madden and R. Mahmood

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

In the extended edition of ‘Lord of the Rings: Return of the King’, the eye of Sauron sees Frodo Baggins at the edge of Mount Doom. Using the Rayleigh criterion the minimum diameter of the Eye’s pupil was calculated to be 3.08 cm to resolve Frodo. We found this diameter to be $\sim 26\times$ the average size of a cat pupil.

Introduction

In Percy Jackson’s famous trilogy adaptation of J. R. R. Tolkien’s novel ‘The Lord of the Rings’, the Dark Lord Sauron’s corporeal form is depicted as a huge all-seeing eyeball atop the Dark Tower. In the extended edition of ‘Return of the King’ the Eye seems to see Frodo Baggins as he is on the edge of the slopes of Mount Doom en route to destroy the One Ring. In this paper we investigated the minimum diameter of the pupil needed for the Eye to resolve Frodo at this position.

Theory

The Rayleigh criterion describes the minimum angle at which two diffraction patterns become distinguishable [1]. This resolution limit is found when the distance between the initial diffraction peaks is half the width of such peaks. In the films adaptation, the Eye of Sauron is depicted as an eye with a vertical slit pupil which can be approximated as a single slit aperture. When observing through such a pupil, the minimum angle, θ_{min} , can be found using the expression

$$\sin \theta_{min} = \frac{\lambda}{d}, \quad (1)$$

where λ represents the wavelength of observed light and d represents the slit diameter. Using the small angle approximate of $\sin \theta \approx \theta$ this expression becomes

$$\theta_{min} \approx \frac{\lambda}{d}. \quad (2)$$

In order to find d , λ and θ_{min} were identified. The human eye is most sensitive to wavelengths at 555 nm [2]. We assumed the eye of Sauron was most sensitive to the same wavelength as spectral sensitivity depends on ocular cones lying on the retina, not pupil shape [3].

The Eye gazed over Middle-Earth from a height over 1,500 m at the topmost tower of Barad-dûr, aka The Dark Tower [4]. Frodo and Sam were described as near the base of Mount Doom when the Eye looks towards them and Frodo appears to collapse.

Figure 1 depicts a schematic of the distances and angles used to find d . The Dark Tower is of height, h , 1500 m, the distance to Mount Doom, D , is 48280 m [5], and Frodo’s height, y , is 1.07 m [6]. We found the angle θ using the cosine rule as shown in Equation 3,

$$y^2 = b^2 + c^2 - 2bc \cos \theta. \quad (3)$$

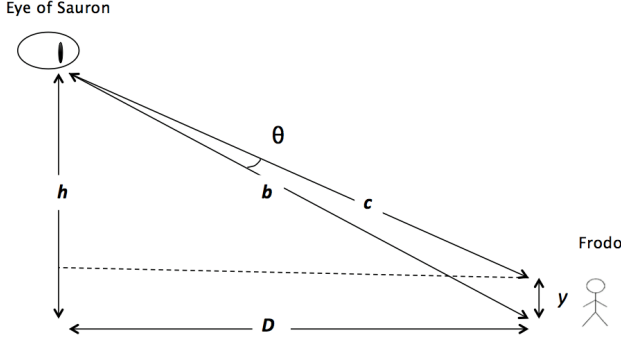


Figure 1: Diagrammatic view of Frodo relative to the Eye of Sauron.

The lengths b and c were found using Pythagoras' theorem. Triangle of sides h , D and b was used to find length b (Equation 4). Triangle of sides $h - y$, D and c was used to find length c (Equation 5).

$$b = \sqrt{h^2 + D^2}. \quad (4)$$

$$c = \sqrt{(h - y)^2 + D^2}. \quad (5)$$

Results

Substituting the equations for b and c into Equation 3 we calculated θ to be 1.80×10^{-5} rad. Using θ as our θ_{min} value and $\lambda = 555$ nm, we rearranged Rayleigh's criterion (Equation 1) for d . This resulted in a minimum slit diameter of $d = 3.08$ cm.

Conclusion

To conclude, we have calculated the minimum diameter for the Eye of Sauron to resolve Frodo when he is at the base of Mount Doom to be 3.08 cm. As the shape of Sauron's Eye is similar to that of a cat's, we compared it with the average diameter of an adult cat's pupil of 1.20 mm [7]. The Eye is found to be $\sim 26\times$ larger than a cat's.

Given that the Eye belongs to a non-human spirit, it may have a very different spectral sensitivity. Animals with vertical slit pupils tend to have a larger spectral range compared to humans which includes UV light[8]. If this was the case for Sauron's Eye, the minimum diameter needed

would likely be smaller than 3.08 cm. The film depicts a very large Eye and pupil which is reasonable as our calculated diameter is a minimum only.

References

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