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A1_02 Moon Light

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Abstract

Scientists appreciate a strong laser but for everyone else, what does a strong laser mean? In this article, we explore how strong of a laser is necessary to see some light from the Moon with the human eye on Earth. We use the luminance threshold of a human eye under ideal conditions to find a minimum power of 8.29×10^{12} W. At this power, we have some unintended results and we must start considering the consequences of our assumptions.

Introduction

In this article, we address how strong an individual laser must be to resolve an image on the surface of the Moon from Earth. There are many variables affecting this, but our main focus is considering a single, powerful laser, whose power we find by considering the minimum number of photons that humans can register as a stimulus.

Theory

We take the conditions at night (in darkness) against a new moon. We use trigonometry to obtain a beam radius, r_{beam}. Taking the closest distance of approach as 357,495 km [1], we subtract the radii of the Earth and the Moon to obtain the distance from surface to surface, $d = 349,837$ km [2]. Thus:

$$r_{beam} = dtan\left(\frac{\theta}{2}\right) \tag{1}$$

Where θ is the divergence of the beam. The power needed to produce this beam can be found by considering the luminance threshold, L_{th}, the minimum level at which light can be detected 50% of the time. This tells us the minimum intensity needed to observe some light. This is in terms of candela (cd), a measure of luminous intensity defined as [3]:

$$I_{\nu}(\lambda) = 683.002 \cdot \bar{y}(\lambda) \cdot I_e(\lambda) \tag{2}$$

Where $I_{\nu}(\lambda)$ is the luminous intensity in cd, $\bar{y}(\lambda)$ is the scotopic (in dark conditions) luminosity function and $I_e(\lambda)$ is the radiant intensity in W sr^{-1}.

To obtain power, take $I_e(\lambda)$ from Eq.2 and multiply it by the solid angle over which the power is distributed. This solid angle is the one subtended by the beam radius on Earth to the Moon.

We assume that the reflection that occurs at the Moon's surface is specular. This means light can be treated like it was reflecting off of a mirror, which means we also assume the Moon's surface is flat and smooth. This is in contrast with diffuse reflection, where a single ray can be scattered at many angles.

Taking specular reflection, the angle of incidence is the same as the angle of reflection, meaning the solid angle subtended from the beam on the Moon to Earth is the same as the one from the beam radius on Earth to the Moon. Thus:

$$P_{obs} = I_e(\lambda) \cdot \Omega \tag{3}$$

L_{th} has units cd m^{-2} (i.e. $L_{th} \times$ area $= I_\nu(\lambda)$), so from Eq.2 and taking Ω as the solid angle subtended by the beam on the moon:

$$P_{obs} = \frac{L_{th} \cdot \pi r_{beam}^2}{683.002 \cdot \bar{y}(\lambda)} \cdot \frac{\pi r_{beam}^2}{r_{beam}^2 + d^2} \tag{4}$$

Substituting in θ and Eq.1:

$$P_{obs} = \frac{\pi^2 L_{th}}{683.002 \cdot \bar{y}(\lambda)} \cdot \left[\frac{d^2 tan^4 \frac{\theta}{2}}{tan^2 \frac{\theta}{2} + 1} \right] \tag{5}$$

For the power of our laser on Earth, we factor in the losses, which we group together as α. Thus:

$$P_{obs} = \frac{1}{\alpha} \cdot \frac{\pi^2 L_{th}}{683.002 \cdot \bar{y}(\lambda)} \cdot \left[\frac{d^2 tan^4 \frac{\theta}{2}}{tan^2 \frac{\theta}{2} + 1} \right] \tag{6}$$

Results

In ideal physiological conditions, the luminance threshold is $L_{th} = 7.5 \times 10^{-7}$ cd m^{-2} [4]. We take $\theta \approx 1 \times 10^{-3}$ rad [5]. We also choose $\lambda = 503$ nm such that $\bar{y}(\lambda) = 1$ (by definition, since it is the most receptive wavelength) [3]. For losses, the Lunar Laser Ranging experiment only detects 1 in 10^{17} photons on good conditions [6], which accounts for factors such as atmospheric losses, the moon's reflectivity, etc, giving $\alpha = 10^{-17}$. This gives $P_{init} = 8.2899 \times 10^{12}$ W.

Discussion

We have made a number of assumptions, implicit in our solution. One was that the drop in power from increasing wavelength would not compensate for the reduced sensitivity of the rods. From Eq.6, taking a value other than $\lambda = 503$ nm would increase the power, which is not what we want if we want a lower bound. Beam divergence could be reduced by taking a large initial beam width, for example, but these considerations would be more suitable for a follow up paper.

The main source of error comes from the assumption of specular reflection at the Moon's surface. One aspect of this is that the reflection occurs on a curved surface, which is another factor that has to be considered. This also affects the solid angle, since the power is not evenly distributed and is spread over a much larger area. If we want to obtain a minimum power, this is acceptable, since consideration of diffuse reflection only increases this.

Conclusion

In reality, the laser is constantly interacting with the atmosphere and with a power this high, it will be imparting the energy equivalent of 1 Little Boy, the nuclear bomb detonated on Hiroshima, every 10 seconds [7]. By evaluating the heat capacity of the atmosphere, we find that concentrating this amount of energy on a small part of the atmosphere will increase its temperature significantly and eventually create a plasma [8].

References

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