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P1_2 Freezing the Oceans

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Abstract
We modelled the Earth's oceans as a series of concentric shells of water. By finding the freezing temperature of saline water at each depth, we found the energy change required to reduce the temperature of each shell to freezing point. By including the energy change needed to cause a change in state, we found that the total energy change required to freeze the oceans is approximately $7.0 \times 10^{23} kJ$.

Introduction

Water covers over 70% of the Earth, most of this water is concentrated in the oceans in the form of saline (salt water). Normally, only a small percentage of this is frozen solid at the poles. We calculated the approximate energy change required to freeze all of the oceans on Earth.

Method

The average depth of the ocean is 3700 m [1]; in order to simplify this problem we modelled the oceans as 7 spherical shells on the surface of the planet, each with a depth of 500 m and found the energy change required to freeze each shell.

Eq. 1 was used to find the volume of each shell:

$$V = \frac{4}{3}\pi((R_E - h_t)^3 - (R_E - h_b)^3), \qquad (1)$$

where R_E is the radius of the Earth, h_t is the depth of the top of the shell and h_b is the depth of the bottom of the shell. As approximately 71% of the Earth's surface is covered by salt water [2], we multiplied this volume by 0.71 and the density of saline water $(\rho, 1030 kgm^{-3})$ [3] to give the mass of each shell. But as h_t and h_b are very small compared to the radius of the Earth, we found that all the shells had approximately the same mass $(1.85 \times 10^{20} kg)$.

We calculated the average pressure across each shell using Eq. 2 [4]:

$$p = p_0 + \rho g h, \qquad (2)$$

where p_0 is atmospheric pressure $(1.01 \times 10^5 Pa)$, ρ is the density of saline water, g is the acceleration due to gravity (assumed constant at $9.81 ms^{-2}$) and h is the median depth of each shell.

To freeze saline water, first the temperature of the water needs to be reduced to freezing point, then the latent heat of fusion needs to be overcome to cause a change of state, from liquid to solid. This is summarised in Eq. 3 [4]:

$$Q = mc\Delta T + mL, \qquad (3)$$

where Q is energy in kJ, m is mass, ΔT is the change in temperature and L is the latent heat of fusion $(334 kJkg^{-1})$ [4].

The freezing point of saline water is dependent on salinity and pressure, as shown in Eq. 4:

$$T_f = a_0 s + a_1 s^{3/2} + a_2 s^2 + bp, \tag{4}$$

where a_0, a_1, a_2 and b are constants, s is practical salinity (g salt per kg water) and p is pressure in decibars [5]. Assuming that salinity remains constant with depth, we used an average ocean salinity of 35 parts per thousand [6] and the pressures calculated using Eq 2, to find the freezing temperature for each shell of ocean.

By comparing the calculated freezing temperature with the average temperature at each depth [7], we found the temperature change required. This temperature change was used with Eq. 3 to find the total energy change required.

Results and Discussion

The table below shows the calculated pressures and freezing temperatures for each shell and the average water temperature at that depth.

Depth (m)	p (MPa)	T_f (°C)	Average temperature (°C)
0-500	2.63	-2.1	10
500-1000	7.68	-2.5	7
1000-1500	12.7	-2.9	4
1500-2000	17.8	-3.3	4
2000-2500	22.8	-3.6	4
2500-3000	27.9	-4.0	4
3000-3500	33.9	-4.4	4

Table 1: Values calculated using Eq. 2 and Eq. 4

These values were used with Eq. 3 to find the energy change. Fig. 1 shows the energy change required to reduce the temperature in each shell down to freezing temperature. This value reaches a minimum at approximately 2500 m, when average temperature is low and pressure is not high enough to cause a large decrease in freezing temperature. However, as mass and latent heat is constant for all shells, we found that the second term in the Eq. 3 dominates and the first term becomes negligible. By including the energy change required to cause a phase change, each shell needs an energy change

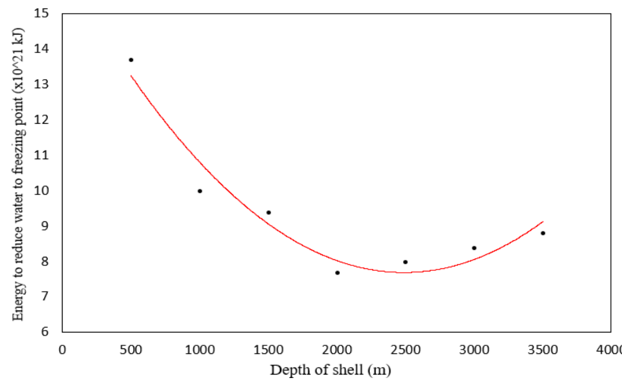

Figure 1: Energy change required to freeze each 500 m shell of ocean

between 9.8×10^{22} and $10 \times 10^{22} kJ$. A total energy change of $7.0 \times 10^{23} kJ$ is required to freeze all the oceans on the Earth.

Conclusion

Overall, we found that the energy change required for the phase change from liquid to solid is much larger than that needed to reduce the shell's temperature to freezing temperature. To improve the accuracy of the result we could use a more complex model that considers different average temperatures and salinities for different places around the Earth.

References

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