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A4_1 Cool Orbit

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Abstract

The Intergovernmental panel on climate change (IPCC) estimates the average temperature of the Earth will rise by 2 K to 4 K between the periods of 1980-1999 and 2090-2099. If the radius of the Earth's orbit was increased, so less radiation from the Sun is received, global average temperatures would drop. We calculated that to negate the temperature increase, the radius of the Earth's orbit would have to be increased to a value between 1.016 AU and 1.032 AU. Using a Hohmann transfer to increase the Earth's semi-major-axis, the Earth's total velocity change was calculated to be between 235.5 ms^{-1} and 465.4 ms^{-1}.

Introduction

The amount of energy the Earth receives from the Sun is inversely proportional to the square of the distance between the two bodies. If the Earth was moved away from the Sun, less radiation would be incident upon it, lowering the equilibrium temperature. We discuss how much the Earth would need to be distanced from the Sun, in order to counter the warming that is projected by the Intergovernmental panel on climate change (IPCC), to occur between the average temperature in 1980-1999 and 2090-2099 [1]. In addition, we will calculate the total change in velocity of the Earth required to achieve this orbital manoeuvre in a single orbit, via a Hohmann transfer.

Temperatures and Orbits

To determine the temperature change, and therefore how much the Earth needs to be moved by, we have taken results from an IPCC report comparing the global average temperature in 1980-1999 to a projected global average in 2090-2099 [1]. Estimates of the Earth's temperature rise range from around 2 K to 4 K, depending on projected emission levels in a best and worst-case scenario [1]. The theoretical model of thermal equilibrium of a planet orbiting the Sun, is derived from the temperature of the Sun and Earth, and assumes they are black body radiators. This model is represented in Eq. (1), where $T_\odot$ and $R_\odot$ are the surface temperature and radius of the Sun respectively, D is the radius of a planet's orbit, T is its surface temperature, and α is its albedo.

$$T = T_\odot (1-\alpha)^{1/4}\sqrt{R_\odot/2D} \qquad (1)$$

From Eq. (1), assuming that the temperature and radius of the Sun, and the albedo of the Earth will remain constant, then the planet's temperature is inversely proportional to the square root of the planet's orbit. Using this proportionality, Eq. (2) can be derived, which shows a relationship between the Earth's original orbital radius (D_0), projected temperature for 2090-2099 (T_0), and the new required orbital radius (D_1) at the desired temperature (T_1).

$$D_1 = D_0 T_0^2 / T_1^2 \quad (2)$$

The equilibrium temperature of the Earth is 255 K [2], significantly lower than the true value of 288 K. This is due to positive radiative forcing, which occurs when a planet experiences an energy imbalance, as more energy is absorbed than radiated by the planet [3]. Radiative forcing is increasing due to both anthropogenic and natural effects, such as industrial addition to greenhouse gases [3]. We have assumed these radiative forcing effects will remain consistent, despite the reduced incoming radiation from the Sun.

We assumed that D_0 is 1 AU, T_1 is 255 K, and T_0 is 257 K or 259 K, for the 2 K or 4 K changes respectively. We have used Eq. (2) to show that for a 2 K decrease in temperature the Earth would need to be 1.016 AU from the Sun, and similarly for a 4 K decrease 1.032 AU from the Sun.

Orbital Transfer

To move the Earth to between 1.016 AU and 1.032 AU, a Hohmann transfer would be the most efficient way excluding gravitational assists [4]. In the following equations for the Hohmann transfer, r_1 is radius of the Earth's initial orbit 1 AU, r_2 is the radius of the final orbit 1.016 AU or 1.032 AU, μ is gravitational parameter of the Sun $132,712 \times 10^6$ km$^3 s^{-2}$ [4] [5], Δv_1 and Δv_2, are the increases in velocity required for the first and second, burns respectively. The Hohmann transfer is made up of two burns, the first accelerating the Earth into an elliptical transfer orbit with a periapsis on the lower orbit, and the apoapsis on the higher orbit [4].

$$\Delta v_1 = \sqrt{\mu / r_1} (\sqrt{2 r_2 / r_1 + r_2} - 1) \quad (3)$$

The second burn accelerates the Earth into a final circular orbit [4].

$$\Delta v_2 = \sqrt{\mu / r_2} (1 - \sqrt{2 r_1 / r_1 + r_2}) \quad (4)$$

The total change in velocity (Δv_{total}) of the manoeuvre is given by Eq.(5) [4].

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 \quad (5)$$

We calculated, using Eq. (5), the transfer to 1.016 AU would require a Δv_{total} of 235.5 ms^{-1}, and to 1.032 AU a Δv_{total} of 465.4 ms^{-1}, assuming the Earth starts in a circular orbit of radius 1 AU. In these calculations, we have assumed the two burns at periapsis and apoapsis of the transfer orbit are instantaneous impulses. This is a reasonable assumption as they would be very short compared to the orbital period of 1 year.

Conclusion

To counteract the predicted effect of climate change until 2090-2099, the Earth's semi-major-axis would have to be increased to at least 1.016 AU and at most 1.032 AU. To achieve the change in orbital size a velocity increase between 235.5 ms^{-1}, and 465.4 ms^{-1} would need to be implemented.

We have not considered further changes to radiative forcing effects that may be encountered after moving, and have only considered the reduction in the planetary equilibrium temperature. The feasibility of accelerating the Earth between 235.5 ms^{-1} to 465.4 ms^{-1} has not been assessed. Additionally the influence on the climate any method of acceleration might have has also not been considered, and depending on the process used, may cause a large discrepancy by furthering the effect of radiative forcing.

References

[1] R. K. Pachauri, A. Reisinger, IPCC Synthesis Report AR4, p. 45. (IPCC, 2007).

[2] `https://scied.ucar.edu/planetary-energy-balance-temperature-calculate` [Accessed 03 October 2017]

[3] `http://www.co2offsetresearch.org/aviation/RF.html` [Accessed 03 October 2017]

[4] D. Vallado, *Fundamentals of Astrodynamics and Applications*, 4th Edition (2013)

[5] D. R. Williams, Sun Fact Sheet, (NASA, 2016)