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P1_6 Escaping a Fire Using a Mattress

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Abstract

This article considers the net energy of a rigid body jumping from a first floor window onto a mattress below, comparing it to the energy needed to break a leg bone. It is found that the final energy of the body after landing is 2739.1J, and calculations show that it takes 385.71J to break two legs. Therefore a mattress alone does not absorb enough energy to avoid injury.

Introduction

In the case of a house fire in which one is trapped in a first floor bedroom, one possible method of escape is to throw a mattress out of the window and jump down on to it. This article calculates the net energy of a man jumping onto a mattress and landing without bending his legs, comparing it to the energy needed to break a bone.

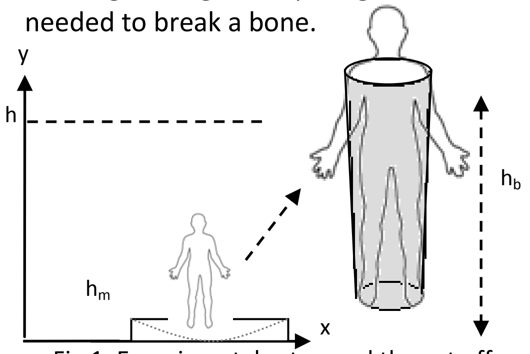

Fig.1, Experimental setup and the cut off cone approximation

Model

This model is based upon a man of height h_b and mass, m of 80kg. Where data on a body is unavailable, the body is approximated to a cut off cone of base radius r_b estimated as 0.1m and top radius r_t of 0.17m [1], as in Fig.1.

This article considers the most potentially damaging situation in which all energy is transferred directly to the bones.

The body hits the ground with a net energy

$$E_{net} = E_{PE\ body} - E_{drag} - E_{PE\ spring} \quad (1)$$

I – Potential Energy of Body

The initial gravitational potential energy of the body at h, the height of the window (Fig.1) is given by

$$E_{PE\ body} = mgh = 2746.8\,J, \qquad (2)$$

calculated with m=80kg, g=9.81m and h=3.5m. The value of h varies from house to house, so this is an approximation.

II – Dissipative (Drag) Energy

The energy dissipated due to air resistance as the body falls through still air is given by [2]

$$E_{drag} = \int_0^h \frac{C_d \rho A({v_o}^2 + 2gy)}{2} dy = \left. \frac{C_d \rho A g y^2}{2} \right|_0^h = 7.46J \quad (3)$$

where C_d is the coefficient of drag, 1.0-1.3 for a standing human [2] (here 1.15 is used), ρ is the density of air, 1.2kg.m^{-3}, h is 3.5m and v_0, the initial velocity of the body, is 0m.s^{-1}. A is the frontal area of the body. To simplify this, the body is approximated to a falling cone, and A is πr_b^2, 0.09m^2. [3]

III – Potential Energy of Springs

Hooke's Law gives the force needed to compress a spring as a function of the spring constant k and the compression distance h_{mc} [4]. The potential energy of a mattress spring is given by

$$E_{PE\ spring} = \int_0^{h_{mc}} ky\,dy = \left. \frac{ky^2}{2} \right|_0^{h_{mc}} \quad (4)$$

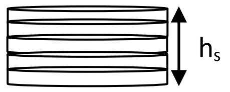

Fig.2, min. spring height, h_s

The mattress used in this model [5] has dimensions 1.88m x 1.35m x 0.22m, with 312 separate steel wire springs, each with 5 turns (and hence 3 active coils) and a wire diameter, d_m=1.8x10^{-3}m [6].

The mattress absorbs the maximum amount of energy by compressing to the greatest possible extent. Assuming the springs compress such that each turn sits on top of the previous, as in Fig. 2, then the minimum mattress height is the minimum spring height, h_s=5xd_m=0.009m. The maximum compression distance h_{mc} is thus 0.22m – h_s = 0.211m.

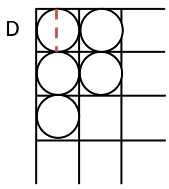

Fig. 3: Approximation of coil diameter

The spring area is found by dividing the mattress area by the number of springs, giving 312 squares of side 0.09m in length.

Assuming the spring diameter D is the same length as the side of a square (Fig.3), then the number of springs, n, compressed by the body's landing is

$$n = \frac{\pi r_b^2}{D^2} = 3.89$$

Considering only a whole number of springs, n=4, giving the number of active coils, n_a=12.

The overall spring constant of the compressed springs can be found using [7]

$$k = \frac{G d_m^4}{8 D^3 n_a}, \quad (5)$$

Where G is the shear constant of steel, 84x10^{9}N.m^{-1} [8] and all other values are as previously defined. Using these, k=12.6Nm^{-1}.

Using this value it is possible to calculate $E_{PE\ spring}$ using (4).

$$E_{PE\ spring} = 0.28J \quad (6)$$

IV – The Total Energy

From (1) and substituting the values from (2), (3) and (6), it is found that the net energy of the body after falling onto a mattress and compressing it to its greatest extent is

$$\begin{aligned} E_{net} &= 2746.8 - 7.46 - 0.28 \\ &= 2739.1\mathrm{J} \quad (7) \end{aligned}$$

The Breaking of Bones

Assume a bone is elastic until the point at which it fractures, then the Young's Modulus, Y and the breaking stress S_B can be used to find the energy needed to break a single bone

$$E_{break} = \frac{A_b l S_B^2}{2Y} = 192.9J, \qquad (8)[9]$$

A_b is the cross sectional area of leg bone = 6x10-4m^2, l is the length of leg bone, taken as h_b/2=0.9m, Y=14x10^{9}Pa, and S_B=1x10^{8}Pa [9]. Hence 385.71J is required to break two legs.

Conclusion

For this model it is found that the net energy of the body is 2739.1J and 385.71J is required to fracture the leg bones. Thus a mattress does not reduce the impact energy of a fall from a first floor window by a great enough amount to prevent a break.

In this article the man lands on his feet, and no energy is absorbed by any other part of the body. Since in reality joints would flex to minimise damage, it would be interesting to find the result with an appropriate landing.

Fully accurate values for the body would make the E_{drag} calculation more reliable.

Here, the size of the mattress springs was roughly approximated, and mattress padding was not taken into account. More accurate values for the springs and the padding would provide a more reliable value for $E_{PE\ spring}$.

References

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