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A2_5 Going Ape. The Galilean Square-cube law in motion pictures.

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Abstract
The article provides a quantitative analysis of the popular film concept of scaling up everyday creatures to enormous sizes. We use the 2005 blockbuster hit King Kong ™ as a specific example. Calculations showed that King Kong's legs would experience less than 2% of their breaking stress while standing, but falling from a 5m height requires Kong to bend his legs throughout the impact for at least 0.02s to avoid breaking them.

Introduction
Since its demonstration in 1638 by Galileo [1], the Galilean Square-Cube law has been changing the way that scientists view nature and evolution. This fundamental concept can be applied to many areas of physics to show that most things have an optimum size for their particular structure.

Enormous creatures that defy the limits imposed by nature have become common in fictional entertainment, particularly motion pictures. It is the purpose of this article to show that the effect of Galilean square-cube scaling should not be neglected in motion pictures.

Analysis
For the purpose of this article, the structural integrity of the giant gorilla in the 2005 blockbuster hit King Kong ™ will be analysed. A typical male gorilla has a mass of approximately 180kg and a height of approximately 1.8m (Free standing, i.e. not using arms for support) [2]. While free standing, we define the load-stress upon the Gorilla's thigh bones as

$$T_G = \frac{W_G}{A_G} \quad (1),$$

where T_G is the load stress, W_G is the total gorilla weight (product of gorilla mass, M_G, and acceleration due to gravity, g) and A_G is the total cross sectional area of thigh-bone bearing this weight.

Consider scaling up the gorilla by n times in all dimensions. An increase in all linear dimensions by n times increases A_G by n^2 times, and the gorilla volume, V_G, by n^3 times. By using the mass density equation we can write the Gorilla's weight as

$$W_G = V_G \rho g \qquad (2),$$

where the mass density, ρ, is assumed constant and g is constant over small distances from the Earth's surface. It follows from (2) that an increase of V_G by n^3 times translates to an increase of W_G

by n^3 times. We can hence define the simple scaling relation between King Kong's cross sectional thigh area, A_K, and his weight, W_K, by the relationships in (3).

$$W_K = n^3 W_G \ , \ A_K = n^2 A_G \qquad (3)$$

Using equation (1) to define the load stress upon Kong's thighs as T_K and using (3) to express W_K and A_K as functions of the original un-scaled quantities W_G and A_G, we obtain

$$T_K = \frac{W_K}{A_K} \equiv \frac{(n^3 W_G)}{(n^2 A_G)} \equiv nT_G \qquad (4)$$

Equation (4) shows that the load stress upon Kong's bones is n times that experienced by a typical male gorilla. Using M_G = 180kg, g = 9.81 m/s^2, and A_G = 0.0039m^2 (the equivalent of twice a human femur [3]) gives $T_K \approx 4.5$ MNm^{-2} if we assume n = 10 (A reasonable first estimate for the size of King Kong). The load stress required to break bone is about 200 MNm^{-2} [4], so in this case it can be seen that Kong could stand without a problem. How would King Kong's legs respond to the impact of a high fall as seen in the movie?

Over small vertical distances, h, we expect air resistance to be negligible. Given an initial falling speed of u = 0 m/s, and the acceleration due to gravity $g = a \approx 9.81$ m/s^2, we use the constant acceleration equation to determine the landing speed of Kong after a fall of h to be

$$v = \sqrt{(u)^2 + 2gh} \equiv \sqrt{2ah} \qquad (5)$$

Assuming Kong's momentum is reduced to zero upon the impact; the ground imparts an impulse and hence increases the load stress upon his legs. This stress contribution is given by equation (6) where Δt is the time of contact during the impact.

$$T_K = \frac{n^3 M_G (v-0)}{n^2 A_G \Delta t} \equiv \frac{n M_G v}{A_G \Delta t} \qquad (6)$$

From equation (6), it can be seen that Δt is inversely proportional to T_K. King Kong should try and make Δt as large as possible (by compressing his legs upon impact) to reduce the load stress upon his thigh bones. At criticality T_K = 200MNm^{-2}, hence using a typical drop of h = 5m in (5), and A_G =0.0014 m^2, n = 10, M_G = 180kg in (6) gives a minimum time of contact during the impact to be $\Delta t \approx 0.02$ s. In the film Kong frequently hits the ground running, making little effort to compress his legs.

Conclusion

Our results show that square-cube scaling is a significant effect when increasing the size of a creature for a motion picture. Though these scaled behemoths may be safe to stand up (Kong's legs bearing less than 2% of their critical load stress while standing upright), they come to a greater risk when engaging in particularly dynamic activity such as leaping off of high objects. For a 5m fall, Kong would need to bend his legs for <u>at least</u> 0.02s to avoid them reaching the critical breaking stress.

References

[**1**] http://www.dinosaurtheory.com
[**2**] http://animals.howstuffworks.com/mammals/ape-info.htm
[**3**] Table 10, $\bar{X}$ for male m-1 midshaft femur diameter,
http://www-personal.une.edu.au/~pbrown3/skeleton.pdf
[**4**] Bone stress against body mass plot, http://fathom.lib.uchicago.edu/2/21701757/