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# A5_1 Power of a Star in an Unreasonably Priced Car 

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#### Abstract

The inaugural Formula E motor-racing championship features electric cars whose batteries are not currently capable of lasting an entire race, meaning drivers must change cars halfway through. This paper investigates the possibility of using solar panels to recharge the battery during the race; thereby ensuring only 1 car per driver is required. Whilst it is established that the car could support a sufficiently large solar array, the required area of $1920 \mathrm{~m}^{2}$ is clearly too large to be of practical use.


## Introduction

The recently created Formula E motor-racing series aims to replicate the excitement of Formula 1 using electric vehicles, thereby enhancing the development of environmentally friendly cars. For the inaugural season, however, the batteries do not have the capacity to power the motor for more than half of the race, meaning that drivers must change to a second car in order to complete the race [1].

In this paper we investigate the possibility of using solar panels to recharge the battery during the race, and calculate the maximum size of solar array that could be supported by the car's body.

## Theory

In order to estimate the size of solar array required to power the cars we must first calculate the total energy that can be supplied to the motor by the battery, assuming the energy transfer is $100 \%$ efficient. The energy, $E_{b}$, supplied by a battery is given by

$$
\begin{equation*}
E_{b}=P_{\text {ave }} t_{\max } \tag{1}
\end{equation*}
$$

where $P_{\text {ave }}$ is the average power delivered by the battery and $t_{\max }$ is the maximum time over which that power can be delivered before the battery is depleted.

The solar array must provide energy to account for the deficiency arising as a result of the total requirement being greater than the battery capacity.

$$
\begin{equation*}
E_{S A}=E_{t o t}-E_{b}, \tag{2}
\end{equation*}
$$

where $E_{\text {tot }}$ is the total energy required to power the car for the duration of the race, and the
energy, $E_{S A}$, generated by a solar array is given by

$$
\begin{equation*}
E_{S A}=p \eta_{s} \eta_{b} I A t \tag{3}
\end{equation*}
$$

In Equation (3), $p$ is the packing factor of the array, $\eta_{s}$ is the efficiency of the solar array, $\eta_{b}$ is the efficiency with which the battery recharges, $I$ is the incident solar flux, $A$ is the area of the solar array, and $t$ is the time over which the array is generating energy. Equation (3) arises as the product of flux, area and time gives the total incident energy, with the other terms accounting for various efficiency factors. The packing factor represents the fraction of the array which is occupied by solar cells.

The maximum area of array that can be supported by the car, $A_{\text {max }}$, can be calculated using the compressive strength of the car's body, $\sigma_{c}$ :

$$
\begin{equation*}
A_{\max }=\frac{\sigma_{c} A_{s}}{P} \tag{4}
\end{equation*}
$$

where $A_{s}$ is the area of the car which is supporting the array and $P$ is the pressure exerted by the array.

## Results

During a race, the maximum power output of the battery is 150 kW [2]. Assuming the cars to be accelerating at full throttle for $75 \%$ of the lap, $P_{\text {ave }}$ is then 112.5 kW . The total duration of the first race was approximately 55 minutes [3], so a battery that can only last half of this time has $t_{\max }=1650 \mathrm{~s}$, resulting in an energy capacity of 186 MJ (from Equation (1)). The total energy requirement will be double this value, as two batteries are fully discharged during the course of the race, meaning that the right hand side of Equation (2) is 186 MJ .



Figure 1(i) (top) - graph showing how the size of the solar array affects the number of laps that can be completed. The dashed line indicates the length of the race, and therefore the size of array needed. Figure 1(ii) (bottom) - graph showing the size of array needed for various battery capacities in order to complete the 25 lap race. The dashed line represents the energy capacity of the battery used in the cars.

Substituting this value for $E_{S A}$ into Equation (3) gives the size of the solar array needed to generate this amount of energy during the course of the 55 minute race to be $1920 \mathrm{~m}^{2}$. This figure has been calculated using a packing factor of 0.9 [4], a solar array efficiency of $18 \%$ [5], a battery charging efficiency of $97 \%$ [6] and with an incident solar flux, $I$, of $186 \mathrm{Wm}^{-2}$ [7]. The value for $I$ is used assuming that the light is incident perpendicularly on the solar array. Figure 1(i) shows how the number of laps that can be completed varies as the size of the solar array is increased, with the dashed line indicating the size of array required in order to charge the battery sufficiently to be able to complete the race.

Figure 1(ii) shows the results of a further investigation in which various battery energy capacities have been considered, and the solar array size needed to complete the race has been calculated for each value.

The body of a Formula E car is made primarily out of carbon fibre [2], which has an ultimate compressive strength of 570 MPa [8]. Estimating the area of car on which solar panels could be placed to be $1.95 \mathrm{~m}^{2}$ (using the nose and rear of the car) gives a maximum weight that can be supported of $1.11 \times 10^{9} \mathrm{~N}$. For a solar panel exerting a pressure of 11.88 Pa [9], Equation (4)
gives a maximum area that can be supported of $9.54 \times 10^{6} \mathrm{~m}^{2}$.

## Discussion and Conclusion

Whilst it has been shown that the carbon fibre body of the car could support a large enough solar array to recharge the battery, the sheer size of array required to produce enough energy in the time frame required means solar power is unfeasible as a method of powering the cars to the end of a race. Furthermore, it has been assumed in these calculations that the solar array would have no effect on the aerodynamics, whereas in reality there would be a significant influence. Moreover, the additional energy required to move the extra mass of the solar array has not been considered, and this would again greatly increase the required size.

Given that such a large solar array would be needed purely to recharge the battery, it follows that using only solar power to drive the car would require an array of vast proportions, approaching an area of $4000 \mathrm{~m}^{2}$.

Indeed, it would appear that the best way to ensure the cars can complete the race distance is to increase the energy storage capacity of the battery. However, having additional cells would increase the mass of the car, and it would therefore not be as straightforward as simply doubling the capacity of the battery.

## References

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