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P4_3 An Extra Minute

M. Szczykulska, J. J. Watson, L. Garratt-Smithson, A. W. Muir

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

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Abstract

This paper presents a case study of a meteor impact which would result in the addition of one extra minute to the period of the Earth's spin. Initial conditions are set and the meteor mass needed to produce this effect is calculated to be 1.54×10^{19}kg. The effect of the collision on the period of the Earth's precession about the spin axis is also calculated and found to be 18 years shorter than the current 26000 year value.

Introduction

This paper presents a situation where the orbital path of an asteroid crosses the orbital trajectory of Earth, resulting in a collision that increases Earth's spin period by 60s. The meteor impacts a location on Earth on the equatorial bulge and acts to oppose both the spin angular velocity of Earth as well as the orbital angular velocity. The equatorial bulge is at an angle of 23.5° to the Earth-Sun ecliptic plane due to the Earth's tilt.

Setting the initial conditions of the collision

The asteroid modelled in this paper is assumed to orbit the Sun. The greatest speed the asteroid can be travelling at relative to the Earth, v_{rel}, is obtained by first working out the maximum velocity an object can possess while still orbiting the Sun at 1AU.

The maximum speed an asteroid can have at 1AU is approximately equal to its escape velocity at that radius. This is calculated by equating the gravitational potential energy (GPE) of the asteroid at this distance from the Sun with the kinetic energy (KE) of the asteroid;

$$GPE = \frac{GM_S m_A}{R_O} = \frac{1}{2} m_A {v_A}^2 = KE \;, \qquad (1)$$

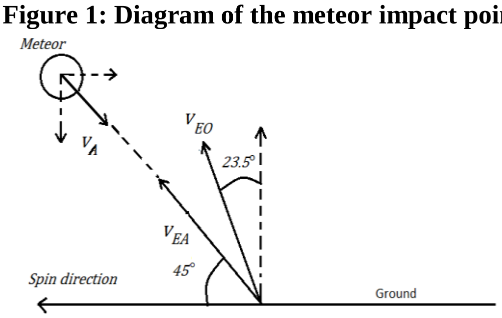
Where G is the gravitational constant, m_A and v_A are the asteroid's mass and velocity respectively, M_S is the mass of the Sun (1.99×10^{30}kg), R_O is the radius of Earth's orbit, 1.5×10^{11}m [1]. The assumption is made that the Sun is the only gravitational force acting on the asteroid.

Re-arranging (1) for the asteroid velocity, v_A, and substituting the correct values for the variables gives v_A = 42.07kms^{-1}.

The most likely angle of the meteor impact to the Earth's surface is 45° [2]. Consulting Figure 1, it is possible to resolve v_A and v_{EO}(Earth's orbital velocity, 29.8kms^{-1} [1]) into their horizontal (x) and vertical (y) components relative to the ground. The x and y components of v_A are in the opposite direction to those of v_{EO}. As a result the two corresponding vector components are simply added together to find the vector components of v_{rel}; v_y and v_x. v_{rel} can then be found using Pythagoras's theorem;

$$\sqrt{{v_y}^2 + {v_x}^2} = v_{rel} = 70.6\text{kms}^{-1}. \qquad (2)$$

Figure 1: Diagram of the meteor impact point.



Alteration of Earth's spin

The change in the spin angular velocity, $\Delta\omega_S$, of the Earth that corresponds to 60s added on to the 24 hour spin period is equal to -5.05×10^{-8}rads^{-1}. The mass of the meteor required to induce this change is calculated by first finding the angular momentum alteration of the Earth at impact;

$$L_{impact} = m_A R_E v_A \cos(45), \qquad (3)$$

where R_E is the radius of the Earth (6371km) and $v_A \cos(45)$ is the component of v_A parallel to the ground (refer to Figure 1). L_{impact} is then equated with the change in angular momentum associated with the required alteration in angular velocity:

$$L_{req} = I\Delta\omega_S\,. \qquad (4)$$

Here $I = \frac{2}{5}M_E{R_E}^2 = 9.69\times10^{37}\ m^2kg$ and is the moment of inertia of Earth (modelled as a sphere of mass $M_E=5.97\times10^{24}$kg) about its central axis. Rearranging the resulting equation for m_A and substituting the correct values gives a required meteor mass of 1.54×10^{19}kg.

Effect on Earth's precession of the spin axis

If the Earth was perfectly spherical the planet would not exhibit precession. However Earth's equatorial bulge results in an uneven mass distribution. An external torque perpendicular to the angular momentum of the Earth's spin is then created by the gravitational pull of both the Moon and the Sun. This results in a change in direction of the spin axis without affecting the magnitude of the spin angular momentum [3].

The force produced by the meteor collision has a component that is perpendicular to the spin angular velocity. However this force is directed through the centre of the Earth and parallel to the equatorial bulge, hence does not affect the net torque on the Earth. Furthermore, the angular velocity of Earth's precession, ω_p, is linked to the spin angular velocity, ω_S. This can be proved through the manipulation Newton's second law for rotation [3];

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}. \qquad (5)$$

Here $d\vec{L}$ is the change in angular momentum over time interval dt and $\vec{\tau}_{net}$ is the net torque on the Earth. Rearranging for $d\vec{L}$ and using the fact the angle the spin axis moves can be estimated via small angle approximation (see Figure 2), gives equation (6) [3];

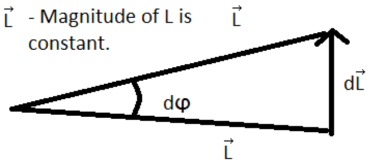


Figure 2: The change in spin angular momentum.

$$d\varphi = \frac{d\vec{L}}{L} = \frac{\vec{\tau}_{net}\, dt}{L}. \qquad (6)$$

Taking the dt term in (6) over to the left hand side results in an equation for ω_P, since this is equal to $\frac{d\varphi}{dt}$. Replacing $\Delta\omega_S$ with ω_S in (4) and substituting this into the re-arranged (6) gives;

$$\omega_P = \frac{d\varphi}{dt} = \frac{\vec{\tau}_{net}}{I\omega_S}. \qquad (7)$$

Equation (7) indicates that ω_S and ω_p are inversely proportional with a constant of proportionality, $c_p = \frac{\vec{\tau}_{net}}{I}$ (assuming the mass distribution on Earth is constant). It is known the current precession period, T, is approximately 26000 years [3]. Therefore the current value of ω_P is calculated via $\frac{2\pi}{T}$ and this is multiplied by ω_S (calculated similarly with T=1 day) giving $c_p = 5.57\times10^{-16}rad^2s^{-2}$. Finally $\Delta\omega_p$ can be calculated using the fact

$$\omega_P + \Delta\omega_p = c_p\left(\frac{1}{\omega_S + \Delta\omega_S}\right). \qquad (8)$$

Via the correct rearrangement and substitutions; (8) gives $\Delta\omega_p = +5.3\times10^{-15}\ rads^{-1}$. This change results in the precession period decreasing by approximately 18 years; which is negligible compared with the original 26000 year value.

Conclusions

Firstly the maximum velocity an asteroid orbiting the Sun can have relative to Earth is approximated to be $70.6kms^{-1}$. Using this initial speed and a 45° impact angle, the meteor mass required to alter the duration of the day by one minute is found to be 1.54×10^{19}kg. Furthermore, the alteration to the planets spin angular velocity would decrease the period of the planets precession by 18 years. Further research could be undertaken into the environmental implications of such an impact.

References

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[3] P.Tipler and G.Mosca, *Physics for Scientists and Engineers*, (W.H. Freeman and Company, New York, 2008), 6th ed., pp. 337-362.