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## P5_12 Thermal Emissions of Projectiles


Peter Hicks, Benedict Irwin, Hannah Lerman

*Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.*



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**Abstract**

This article investigates estimating the velocity and kinetic energy of a projectile, based on the wavelength of radiation emitted due to frictional heating caused by atmospheric drag. An 81kg sphere of tungsten emitting light with a wavelength of approximately 480nm is found to have a velocity of 90km·s$^{-1}$ and hence kinetic energy of 330GJ.


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### Introduction

In the table top miniature war game, Warhammer 40,000 and its popular video game counterpart, Dawn of War, the race “Tau” utilise powerful weapons known as railguns. These railguns are depicted as leaving a residual blue trace in the air [1]. This article discusses estimating the energy of such a weapons projectile, under the assumption that this blue light is due to thermal emission of radiation, caused by the frictional heating of the projectile as it moves through the atmosphere, leaving a streak of blue light.

### Projectile

The projectile is assumed to be made from tungsten as the element is used in munitions due to its high density, melting point and boiling point [2]. It is likely that a technologically advanced race such as the Tau use a different material with more suitable properties, but tungsten should make a good approximation. The projectile is modelled as a sphere of radius 10cm, giving a mass of 81kg [2], which corresponds to approximately 440mol. It is assumed that a high energy is required to make the projectile radiate blue light, and therefore it must be moving at hypersonic speeds through the atmosphere to generate the frictional heating energy.

### Energy associated with blue light

The temperature at which bodies radiate a given peak wavelength of light is given by Wien’s displacement law:

$$\lambda_{peak}T = b, \qquad (1)\ [3]$$

where $\lambda_{peak}$ is the peak wavelength emitted, $T$ is the temperature and $b$ is a constant, approximately 2.9×10$^{-3}$m·K. Cyan light with a wavelength of 480nm gives a temperature of approximately 6000K. Although this temperature is above the boiling point of tungsten at atmospheric pressure, 5828K [2], it is assumed that the high pressures involved with hypersonic shock fronts are sufficient to raise the boiling point above 6000k.

The energy, $Q$, required to raise the projectile by a temperature, $T$, is given by

$$\Delta Q = NC_p\Delta T, \qquad (2)$$

where $N$ is the number of moles in the projectile, and $C_p$ is the specific heat of the projectile in J·mol$^{-1}$·K. However, since $C_p$ is a function of temperature, to find $Q$ we must integrate by taking the equation to the limit where $\Delta\rightarrow 0$. The specific heat of solid and liquid Tungsten is given by the Shomate equation:

$$C_{p_n} = A_n + B_n t + C_n t^2 + D_n t^3 + E_n t^{-2}, \qquad (3)[4]$$

where $t = \frac{T}{1000}$ and the constants $A, B, C, D$ and $E$ have a different value for each $n$, corresponding to the temperature ranges:

- $n = 1$ for $298K < T < 1900K$(solid)
- $n = 2$ for $1900K < T < 3680K$(solid)

- $n = 3$ for $3680K < T < 6000K$(liquid)

Analysis shows the constants $B, C, D$ and $E$ are negligible for $n = 3$, and therefore $C_{p_3} = A_3 =$ constant.

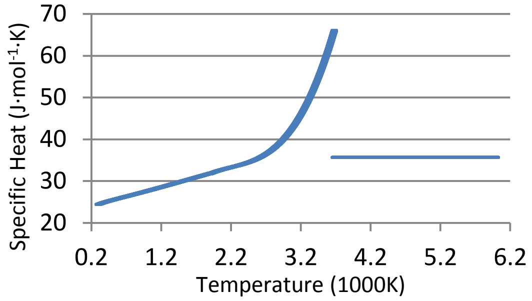


*Figure 1; graph of specific heat with temperature.*

Performing the standard integral

$$Q = 1000N \int_{298}^{6000} t\, C_{p_n}(t)\, dt,$$

yields $Q \approx 292MJ$. However, this is not the total energy required, as the latent heat of fusion must be taken into account for the phase change from solid to liquid. For tungsten this is 0.035MJ·mol$^{-1}$ [2], resulting in an additional 16MJ of energy. Therefore the total energy required to heat the projectile from 298K to 6000K is 308MJ.

**Aerodynamics**

Assuming the drag equation still holds at hypersonic velocities, the force of drag on the projectile, $F_{drag}$, is

$$F_{drag} = \frac{1}{2}\rho v^2 C_d A, \qquad (4)$$

where $\rho$ is the density of the atmosphere, $v$ is the velocity of the projectile, $C_d$ is the coefficient of drag and $A$ is the cross-sectional area of the projectile orthogonal to the direction of movement. The atmosphere is assumed to be similar to Earth's, and hence has a density of 1.217kg·m$^{-3}$ [5]. At Mach numbers and temperatures of 11 to 60 and 2500K to 9000K respectively, the coefficient of drag on a sphere is approximately 2.0 [6].

It is assumed that this force of drag is doing work to the projectile by heating it up. The work done, $W$, is given by

$$dW = \boldsymbol{F}_{drag} \cdot d\boldsymbol{l}, \qquad (5)$$

Where $l$ is the distance over which the drag force is acting. Taking $\boldsymbol{F}$ and $\boldsymbol{L}$ to be parallel, Equations (4) and (5) are combined to give

$$v^2 = 2\frac{dW}{dl}\frac{1}{\rho C_d A}, \qquad (6)$$

**Results**

It is assumed that the projectile is heated to 6000K by drag forces within the last metre of the barrel, where it is approximated that the railguns acceleration on the projectile cancels out with the drag force deceleration, meaning a constant velocity over the distance, such that it can be approximated that $\frac{dW}{dl} = \frac{W}{l}$. Applying this assumption to equation (6) and setting $W = Q$ results in a velocity of approximately 90km·s$^{-1}$. This gives the 81kg projectile a kinetic energy of 330GJ.

**Conclusion**

The velocity of the projectile is extremely high, although not relativistic. This is an expected result and appears reasonable. Although it took 308MJ to heat the projectile to a temperature where it would glow blue, this is still three orders of magnitude less than the total energy of the projectile, and as such drag forces during flight should not significantly reduce the stopping power of the weapon.

This article is limited by the assumptions that heat is distributed evenly across the projectile, no heat is transferred to the atmosphere and the drag equation holds at hypersonic velocities. Future studies might want to investigate this further. It would also be interesting to see whether the nitrogen and oxygen in the atmosphere is heated significantly enough so as to produce additional light.

**References**

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