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P4_7 Make a "Brake" for It!

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Abstract

The paper investigates the plausibility of using a solar sail to reduce the orbital radius of a satellite around Mars instead of the usual methods. This is done by calculating the surface area of sail needed to lower the orbital radius from 3657km to 3517km by deploying the sail for a quarter of the orbit. The resulting surface area is 0.55 km^2, which is impractical as a replacement for current methods.

Introduction

Mars, being one of Earth's closest neighbours, has long been a subject of many satellite observations and rover landings. One of the main stages in these missions is to adjust the orbit of a satellite so that it is in the ideal location to make the desired scientific measurements. Orbital corrections or a reduction in orbital radius can be made through the firing of on-board rockets [1]. Aerobraking can then also be used to provide long-term corrections or to further reduce the orbital radius and is achieved using the frictional force of the Martian atmosphere [2]. Using rockets obviously has an associated fuel requirement, so it would be beneficial to find another method of changing the orbit of a satellite relative to Mars over a short time period. We investigate the use of a "solar sail", where radiation pressure from the Sun provides a braking force, to reduce the orbital radius of a Mars satellite.

Discussion

The orbital change that we are attempting to achieve is shown in figure 1. Note that this scenario is not that which follows a typical orbital insertion, where the initial orbit is highly elliptical. The solar sail would be deployed for a quarter of the initial orbital period, T_1, so that it passes through the point at which it becomes perpendicular to the direction of the Sun.

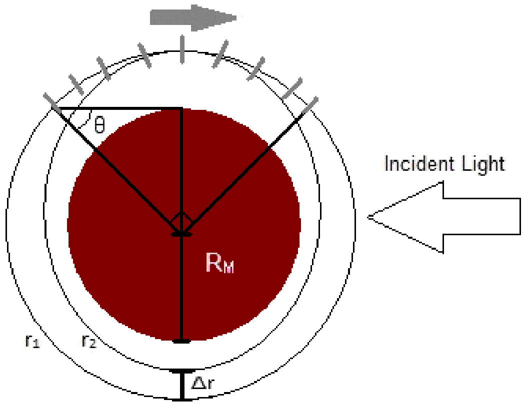

Figure 1 – Initial and desired orbits. R_M is the radius of Mars, r_1 is the initial circular orbit, r_2 is the desired elliptical orbit and Δr is the change in orbital radius. The solar sail is depicted in grey and is shown to be deployed for a quarter of its orbit at a varying angle θ to the incident sunlight.

We assume an initial altitude of ~260km and a desired altitude, after braking using the solar sail, of ~120km [3]. Since Mars has a radius of 3397km [4], this gives values for r_1 and r_2 of 3657km and 3517km respectively.

The force exerted on the sail due to solar radiation pressure, F, is given by,

$$F = \frac{I(1+R)A\,\overline{\cos(\theta)}}{c} \quad , \qquad (1)$$

where I is the solar intensity at Mars, which is obtained by multiplying the solar constant (=1360Wm^{-2}) by r_E/r_M, where r_E and r_M are the orbital radii of Earth and Mars respectively. Using values of orbital radii of 1AU and 1.5AU

[4] gives a value of I=604 Wm^{-2}. R is the reflectivity of the sail (R=1 as we assume that the sail is perfectly reflecting), A is the surface area of the sail and c is the speed of light in a vacuum. $\overline{cos}(\theta)$ is the average value of cos(θ) integrated between ±π/4 and then divided by the θ-range π/2, which accounts for the angular dependence of the force as the sail rotates with respect to the incident light. The sail is assumed to be rotated so that the force will always act to oppose the motion of the satellite. The value of $\overline{cos}(\theta)$ used in our calculations is 2√2/π.

The change in orbital period between the two orbits can be neglected as, through the application of Kepler's 3rd Law, it can be shown that $T_1 \approx 1.04 T_2$, where T_2 is the period of the lower orbit. The sail will therefore travel a distance, x = 5744km during the time that it is deployed.

Therefore the work done on the sail, W, in this time is,

$$W = \frac{I(1+R)A\,\overline{cos}(\theta)x}{c} \quad . \qquad (2)$$

To find the area of sail required to reduce the radius of the initial orbit to give the desired elliptical orbit perigee r_2, we calculate the change in orbital energy between the two orbits [5]. This is the change in the sum of potential and kinetic energy for each orbit and is given by,

$$E = \frac{GMm}{(r_1+r_2)} - \frac{GMm}{2r_1} \quad , \qquad (3)$$

where G is the gravitational constant, M is the mass of Mars (6.4×10^{23}kg)[4] and m is the mass of the orbiting satellite, taken here to be 100kg.

Equating (2) and (3) and rearranging for A gives,

$$A = \frac{cGMm}{I(1+R)\,\overline{cos}(\theta)x}\left(\frac{1}{(r_1+r_2)} - \frac{1}{2r_1}\right). \qquad (4)$$

By substituting in the values given above, the area of sail required to reduce the orbit in one quarter orbit is 0.55 km^2.

Conclusion

The surface area of sail found above is equivalent to the surface area of 440 Olympic-size swimming pools, or 140 times larger than the surface area of the International Space Station solar panel array [6]. This size of sail is obviously too large to serve as a practical replacement for rocket-based orbital manoeuvres. To successfully employ this method of orbital reduction, a smaller sail would have to be used over several orbits to achieve the same result but over a much longer time period. In our model, we have taken an average of the radiation pressure force and assumed that it is applied at one point in the orbit. If the effects of multiple orbits were to be considered, this assumption would become increasingly inaccurate so a more realistic model would have to be created in further studies.

References

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