P1_4 Super(conducting) Cars: Levitation

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Abstract
This article considers using the phenomenon of the Meissner effect in superconductivity to levitate a vehicle above the ground. A model consisting of two parallel wires with opposing currents produces a magnetic field upon which the vehicle levitates. It was found that the power needed to keep the vehicle at 1.75m (the height which minuses the power required) above the ground was 1.88TW. This power is analogous to the overall electrical power consumption of the world - showing that this type of transportation is not feasible with our current technological achievements.

Introduction
The main problem with current forms of travel is that vehicles have to use energy to overcome the force of friction, but a vehicle that levitates would eliminate this problem. This paper tries to outline the possibilities of levitating a vehicle using superconductivity.

When a superconductor (SC) is cooled below its transition temperature in an applied magnetic field, the flux through the material becomes zero and it can be said to become a perfect magnet [1]. The SC is shielded from the applied field due to a flow of screening currents on the surface of the material. The high magnetic susceptibility enables the SC to levitate in the magnetic field. If the SC is displaced, the lines of flux excluded from the material are compressed and this increases the magnitude of the external field; thus a restoring force is exerted on the body.

Model
In this model, the magnetic field is produced by two current carrying wires (see Figure 1). The two wires hold currents in opposing directions so that the magnetic field is at a maximum at the equidistant point. The vehicle is modelled as a superconducting plate on its base of dimensions 1m x 1m x 0.1m (L x W x D) on which a shell is mounted (designed to hold passengers).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure1}
\caption{Two wires carrying current in opposing directions. The superconducting plate (black) is located at the base of the vehicle (blue) and is levitating in the positive z-direction. The vector r is the radial distance of the magnetic field in the z-y plane.}
\end{figure}

Equations
The lift force on the vehicle, $F_L$, is given by [1,2],

\begin{equation}
F_L = -\mu \cdot \nabla B = -\left(\dfrac{xV_B}{\mu_0}\right) \cdot \nabla B,
\end{equation}

where $B$ is the magnetic field, $\mu$ is the magnetic moment, $V$ is the volume of the superconducting plate, $\chi$ is the magnetic susceptibility ($\approx$ -1 in SC [1]) and $\mu_0$ is the permeability of free space. This can be rearranged to give (as the force in the z-direction is opposing gravity - see Figure 1);

\begin{equation}
F_L = \dfrac{VB}{\mu_0} \dfrac{\partial B}{\partial z}.
\end{equation}
The gradient in $B$ and the magnitude in $B$ have been calculated using Ampère’s law in circular co-ordinates (as the system is symmetrical about the x-axis) and multiplied by a factor 2 since there are two wires:

$$B = \frac{\mu_0 l}{2\pi r} \frac{1}{r} = \frac{\mu_0 l}{\pi \sqrt{(x^2+y^2)}} \quad (3)$$

$$\frac{\partial B}{\partial z} = -\frac{\mu_0 l}{\pi} \frac{z}{(x^2+y^2)^{3/2}} \quad (4)$$

The force in equation (2) must exceed $F_g$ (=mg) and therefore substituting Equation (3) and (4) into Equation (2) and rearranging for the current gives,

$$I^2 = \frac{mg\pi^2}{V\mu_0} \frac{(x^2+y^2)^2}{z}, \quad (5)$$

where $m$ is the total mass of the car and $g$ is the acceleration due to gravity (9.8ms$^{-2}$). The value of $z$, the elevation of the car, which minimises the current required to keep the car elevated at this height can be found by differentiating Equation (5) with respect to $z$ and setting to zero. Such a calculation yields a minimising height of $z = 1.75m$, given $y = 2m$ (the mid-point between the two wires). The volume, $V$, of the superconducting plate has been assumed to be 0.1m$^3$ with a density of 5661kgm$^{-3}$ (from the dimensions given in [3]) so that its mass is 566kg. The overall mass, $m$, of the vehicle has then been assumed to be of order 10$^4$kg with the addition of passengers and structure. This gives the current required to keep the vehicle at 1.75m above the ground as 4.7MA.

This current can be used to calculate the electrical power, $P$, required to sustain this magnetic field [4];

$$P = I^2 R = I^2 \frac{\rho l}{A} \quad (6)$$

where $I$ is the current, $R$ is the resistance in the wire, $\rho$ is the resistivity of the wire (assumed to be copper with 1.68 x10$^8$Ohm m [4]), $L$ is the length of the wire (taken as 160km - the approximate distance from Leicester to London as this makes our model suitable for day-to-day travel), and an assumed radius of 0.1m, (giving an area $A$ of 0.03m$^2$). This gives the power required to sustain the wire current to be 1.88TW.

**Conclusion**

The power calculated above is analogous to the world electrical power consumption (=2TW) [5] questioning the feasibility of such a system.

This model does not take into account the limitations of superconductivity i.e. the critical field and critical temperature. If these parameters are exceeded the SC is no longer in a superconducting state and the model breaks down. On top of this, a consideration of how the use of superconducting wires would alter the required power could be investigated because superconducting wires allow currents to flow with a negligible resistance.

This paper has only discussed the theoretical consideration of a superconducting car, but to accomplish these results the practicalities must also be analysed. One issue is due to the Lorentz force pushing the two wires apart. The wires would therefore need to be constriction by structurally fixing them in place [4]. To be able to use this as a proper form of transportation, the mechanism for starting, stopping and turning will also need to be considered, as well as the maximum velocity limitation due to drag effects (this will become important if we are to assume these vehicles can reach high velocities).

**References**


