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P4_2 Beam Me Up

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Abstract

The effects of sending a laser powered platform to low Earth orbit are investigated, considering the effects of drag, attenuation by the atmosphere and divergence of the laser beam. It is found that due to the effects of air drag and attenuation, a 1000kg platform could not be accelerated to an altitude of 200km, requiring approximately 8.7×10^{20}W to maintain a velocity of 100ms^{-1}. The theory allows for the elevation of a platform using more feasible laser power when drag and attenuation effects are removed, i.e to lift off from the surface of planets with no atmospheres; however it is unfeasible to extend this argument to rocket masses.

Introduction

The Mars rover received worldwide news coverage in August 2012 as it touched down. Future missions might be made more viable by reducing the fuel requirements to reach orbital altitudes. In the following sections, we determine the power that would be required for an arbitrary, relatively low mass 1000kg platform to maintain 100ms^{-1} at altitude of 200km (approximately low earth orbit altitude) above the Earth's surface, and extend the argument to elevating massive loads such as a rocket.

Theoretical Model

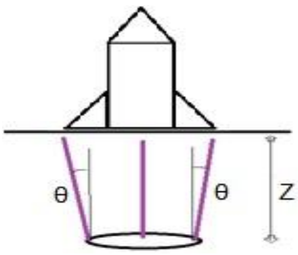

Figure 1: Model of the proposed platform

The theoretical setup is shown in figure 1. *Z* is the altitude of the platform, ϑ is the beam divergence and *D* is the aperture diameter. We start by assuming that all the laser emission is normal to the surface of the platform. This is justified by considering the divergence of light of wavelength λ, through a circular aperture as described by the Rayleigh criterion [1],

$$\theta = \frac{1.22\lambda}{D}. \quad (1)$$

For a beam of wavelength 785nm, the divergence through a 1m diameter aperture is equal to 958×10^{-9} degrees and so can be neglected. Also assumed is that the mass acts from the centre of the platform, so as to avoid torques which may send it into rotation. The radiation pressure acting on the rocket is given by [1],

$$P_{rad} = \frac{I}{c} = \frac{F}{A} \quad , \quad (2)$$

where P_{rad} is the radiation pressure, *I* is the intensity of the light, *c* is the speed of light and the ratio of force to area *F/A* is the definition of pressure. Multiplying through by *A*, we see that the upward force on the rocket by the laser is given by,

$$F = \frac{U}{c}(1 + R) \quad , \quad (3)$$

where *U* is the laser power, and we have introduced the parameter *R*, the reflectivity of the surface. This considers the additional momentum gained by the platform as a photon is reflected from it. A perfectly reflecting surface achieves twice as much momentum from a photon strike than a non-reflecting surface.

Considering the effect of the Earth's gravitational acceleration and that of the drag force experienced by the platform as it travels through the atmosphere, the equation of motion of the platform is:

$$m\frac{dv}{dt} = \frac{U(1+R)}{c} - mg - \frac{1}{2}\rho A C_D v^2, \qquad (4)$$

where m is the mass of the platform and contents, g is the gravitational field strength, A is the cross sectional area of the platform, v is the upward velocity and C_D is the drag coefficient ≈ 0.8[2]. We consider a platform travelling at a constant speed of 100ms^{-1} and the power required to maintain this velocity. The acceleration term of (4) is set to zero and the equation is rearranged for U,

$$U = \frac{c\left(\frac{1}{2}\rho A C_D v^2 + mg\right)}{(1+R)} \quad . \qquad (5)$$

Since density changes as the altitude is increased, we define a mean value of air density as,

$$\frac{1}{Z_a}\int_0^{Z_a} \rho(z)dz = \bar{\rho} = \frac{H}{Z_a}\rho_0\left(1 - e^{\frac{-Z_a}{H}}\right), \qquad (6)$$

where H is the scale height of the atmosphere ≈ 8.4km [3], Z_a is the altitude, and ρ_0 is the surface air density[3] . This term is negligible at higher altitudes due to the exponential decay of air density but is included here for completeness. Substituting this into (5) for a perfectly reflecting platform (with R equal to 1) of radius 2m yields a required laser power of 1.9TW to maintain 100ms^{-1} an altitude of 200km. Initially, the platform is at rest on the surface, with zero drag force. By substitution into (4), a laser power of 1.9TW would thus provide an initial upward acceleration of 2.18ms^{-2}, tolerable by humans. At 200km, g has weakened to a value of 9.36ms^{-2} [3]. This is neglected in our calculation as the reduction in required power is marginal. We also consider the attenuation of the laser signal by the atmosphere. The power term in (4) is multiplied by $\mathrm{e}^{-\sigma z}$ to account for this effect, causing the received intensity to decay exponentially with altitude. While the effect is wavelength dependent, we have approximated a value for the attenuation coefficient σ of 0.1km^{-1} [4]. Taking this into account, the laser power required to sustain 100ms^{-1} increased to 8.7×10^{20}W.

Will it Work for a Rocket?

Assuming a total mass of 2.04×10^6kg[5], the most powerful artificial laser beam to date, tested at approximately 5×10^{12}W)[6], is insufficient to allow the shuttle to leave the surface. Elevating a rocket of the mass stated would require a power output of 3×10^{15}W. This is a minimum value obtained by setting v equal to zero in (4), neglecting the added power required to overcome drag and assuming a perfectly reflecting platform. In short, current lasers are several orders of magnitude too weak to accomplish this.

Conclusion

The feasibility of elevating objects to a low Earth orbit altitude was found to be implausible with laser powers in the terawatt range, even for low mass platforms, unless attenuation effects are neglected. Also neglected here are the consequences of a perfectly reflecting platform at ground level. The returned laser power would need to be dispersed by some mechanism; possibly by use of a reflective attachment to the platform, diverting the laser light at a cost of reducing upward momentum. Future work might investigate this further.

References

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