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P4_1 High Time

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Abstract

The paper attempts to determine the differences in the passage of time experienced over an 78.5 year human life span on the surface of earth and atop of the highest peak of Everest, using a more terrestrial setting than that used in the 2011 paper entitled *A Relative on Olympus Mons*. The effects of general and special relativity are investigated. The theory of general relativity shows a difference of 2.38 milliseconds. The effects of special relativity produce a time dilation 3 orders of magnitude lower, clocking in at 0.008 milliseconds.

Introduction

Sir Edmund Hillary scaled the highest peak of Everest in 1953[1]; a monumental feat of endurance, determination and skill. Upon completing the climb, Sir Edmund achieved an altitude of approximately 8.8km. Such a difference in altitude relative to ground level causes the time dilation effects of both general relativity (resulting from the difference in the Earth's gravitational potential well), and special relativity (caused by the mountain top rotating faster around the earth to maintain the same angular frequency as the surface) to become apparent over extended periods. This effect was investigated for altitude differences on the Martian surface in a previous edition of the journal[2]; however this effect is examined here for a more terrestrial based setting.

Discussion

General relativity predicts that the mass of the Earth curves space time. The outcome of which is that the stronger the gravitational field at a particular radius, the greater the separation between events in time. The effects of general relativity are first examined. This predicts that the time between events as measured by observers both at sea level and atop of Everest will be longer at higher altitudes. The equations governing this effect are,

$$T_e = t_\infty \sqrt{1 - \frac{2GM}{rc^2}} \quad , \qquad (1)$$

$$T_m = t_\infty \sqrt{1 - \frac{2GM}{(r+\Delta r)c^2}} \quad , \qquad (2)$$

where T_e and T_m are the measured proper time differences between two events as measured at seas level and at the summit of Everest respectively, t_∞ is the time between events at an infinite distance, G is the universal gravitational constant, M and r are the mass and radius of the Earth respectively, Δr is the height of Everest above sea level and c is the speed of light in a vacuum [3].

We now go on to calculate this effect over a 78.5 year period (the average human lifetime)[4]. By dividing equation (1) by (2), we calculate a ratio for the time difference at the two positions,

$$\frac{T_e}{T_m} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} \left(1 - \frac{2GM}{(r+\Delta r)c^2}\right)^{-1/2} . \qquad (3)$$

Note that the right hand sides of the bracketed terms in (3) are much smaller than 1. We can therefore use the binomial expansion to eliminate the powers from (3), and expand out the result,

$$\frac{T_e}{T_m} = \left(1 - \frac{GM}{rc^2}\right)\left(1 + \frac{GM}{(r+\Delta r)c^2}\right) \quad . \qquad (4)$$

Rearranging (4) produces a final result of the form,

$$\frac{T_e}{T_m} = 1 + \frac{GM}{c^2}\left(\frac{1}{r+\Delta r} - \frac{1}{r}\right), \qquad (5)$$

where the product of the second terms in the brackets of (4) are negligible. Substituting an Earth mass of $5.97 \times 10^{24}\mathrm{Kg}$,[5] radius of $6.371 \times 10^{6}\mathrm{m}$[5] and the height of Everest $\Delta r = 8.848 \times 10^{3}\mathrm{m}$ [6], the ratio of this time difference is 1 - 9.6×10^{-13}. If we multiply this figure by the 78.5 year period assumed earlier, we see that in an average human lifetime, a person living atop of Everest would experience approximately 2.4 milliseconds less than the fortunate individual to have lived cosily on the surface.

Incorporated now are the effects of special relativity. We can use Einstein's postulate that the speed of light is independent of the frame of the observer to deduce the differences in the measured time between two events, as a result of the different travel speeds. The peak of Everest is 8.8km farther out from the centre of the Earth than its surface so it must rotate around the Earth's axis more quickly, as described by the velocity angular speed relations,

$$v_e = r\frac{2\pi}{P} \quad , \qquad (6)$$

$$v_m = (r + \Delta r)\frac{2\pi}{P} \quad , \qquad (7)$$

where *P* is the orbital period of the earth and v_e and v_m are the rotational velocities at the Earth's surface and at Everest's summit respectively. We assume for simplicity that Everest rotates with the same velocity as the equator and neglect the velocity difference brought on by the Earth's rotation about the Sun. The time between events in a proper, stationary frame ΔT_0 (the Earth's surface or the top of Everest, ΔT_e or ΔT_m respectively), and a frame moving with velocity *v* are related by [5],

$$\Delta T = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\Delta T_0 \quad . \qquad (8)$$

Substituting v_e and v_m into (8) and taking the ratio of the results yields a relation for the effects of special relativity analogous to (5), where the positive term indicates that the effect now reduces the time between events on Everest.

$$\frac{\Delta T_e}{\Delta T_m} = 1 + \frac{2\pi^2 \Delta r}{P^2 c^2}(2r_e + \Delta r) \quad . \qquad (9)$$

Substituting the appropriate values into (9) produces a time dilation effect of 0.01 milliseconds over a 78.5 year period. The net effect is a 2.4 millisecond discrepancy in the perceived passage of time over an average human lifetime.

Conclusion - It's About Time

We have investigated the effects on the passage of time associated with spending a life time at high altitude. While it would appear to shorten one's life, relatively speaking it is shown that approximately 2.4 milliseconds less time will pass for an observer at altitude relative to one at ground level over an entire lifetime.

References

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