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A2_4 Doing whatever a spider can

M. Bryan, J. Forster and A. Stone

Department of Physics and Astronomy, University of Leicester. Leicester, LE1 7RH.

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Abstract
In Spiderman 2 there is a scene in which Spiderman stops a runaway train using his webbing to provide a counter-force. Using the information available this paper examines the material properties of the webbing under these conditions and finds the Young's modulus to be 3.12GPa, a reasonable value for spider silk.

Introduction
In the early sixties Marvel Comics first introduced Spiderman; a superhero with the abilities and scaled strength of a spider. In a recent movie incarnation, Spiderman has the ability to sling webs from spinnerets located in his wrists. These webs have been shown to be capable of taking great amounts of strain, and have displayed a high level of adhesiveness. Arguably the greatest test of these webs is found in the 2004 movie, Spiderman 2; wherein Spiderman manages to bring a runaway train to a stop by sticking multiple webs to adjacent buildings, and bracing himself on the front of the train until it comes to a rest just before dropping into a river [1]. In this paper we attempt to model the forces upon the webbing in such a situation, and compare it to measured values of the Youngs modulus and yield strengths of real spider's web.

Model Parameters
The train we see in the movie is made up of four R160 New York City Subway cars [2], each of mass 38,600kg, with room for 246 people per car. In the scene the cars are all very crowded, so we can assume a maximum occupancy of 984 people across the train. Using an average weight of 70kg per person, this brings the combined weight of the train and passengers to $m_T \approx 2 \times 10^5 kg$. Assuming that the runaway train is travelling at maximum speed, the given velocity being $v_T = 24.6 ms^{-1}$, we can subsequently work out the momentum of the train at full speed to be approximately $P_T \approx 5 \times 10^6 kg.m.s^{-1}$. Using this value, the retarding force applied to the train can be determined by how long it takes for the train to come to rest after the webs are attached. By timing the scene from the attachment of the first web this is found to be $\Delta t = 50s$. For the majority of the time the train is also still attached to powered rail, meaning that it will be pushing back against the restoring force of the webs. The R160 subway car when powered has a maximum acceleration of about $1.12 ms^{-2}$, which gives us a driving force of $F_T = 2 \times 10^5 N$. Since we know that the restoring force of the webs overcomes this, and assuming that the force of rolling friction between steel rails and lubricated steel wheel can be neglected (coefficient of rolling friction $\approx 0.001 - 0.002$ [3]), the force that Spiderman's webs exert on the train can be calculated to be:

$$F_R = -(F_T + \frac{(m_T v_T)}{\Delta t}) = -3 \times 10^5 N \quad (1)$$

assuming that the acceleration is constant over the whole distance.

Calculating the tensile strength of the webs
At the end of this scene, the train is hanging at rest just over the end of the track, but is unpowered, implying that the restoring force should be more than enough to propel the train backwards. As this does not happen, and considering that the webs, upon release, only restore a short distance backwards, it can be safely assumed that they have been strained beyond their yield strength, and are at approximately their elastic limit when still held in place. By calculating the tension in the webs, we can therefore, calculate the stress, strain, Young's Modulus and tensile strength. In figure 1 the webs either side of the train are at different angles to the train, due to the adjacent buildings being different distances from the train when the webs were attached. This means they must be under different tensions, where $T2 > T1$; meaning that, since neither web seems capable of restoring, T1 is a more reliable approximation for the elastic limit. To determine the tension we need to determine angles θ and ϕ. Firstly, we must use an

approximation of the distances from the train to the walls to which the webs are attached; these will be y_1 and y_2, which are about 15m and 18m respectively. Knowing the initial velocity and the rate at which the train decelerates, it is easy to determine x, the distance which it covers after the webs are attached:

$$x = 615m = v_T \Delta t - \frac{1}{2}\frac{v_T}{\Delta t}\Delta t^2, \tag{2}$$

assuming that the acceleration is constant. Having determined this the angles are found to be $\theta = tan^{-1}\frac{y_1}{x} = 1.39°$, and $\phi = tan^{-1}\frac{y_2}{x} = 1.68°$. Since the system is in rest in all directions, we can calculate that:

$$F_R = T_1(\cos\theta + \sin\theta\cot\phi), \tag{3}$$

which can be solved to show that $T_1 = 1 \times 10^5 N$. To determine the stress through the webbing at this point we need the cross-sectional area. At the beginning of the manoeuvre, we see that Spiderman actually slings eight webs either side of the train, each approximately 5mm in diameter judging by close examination of the web held in his hands (Nb the cross-sectional area will change slightly with extension, but would be impossible to estimate). Modelling this as a single rope made up of eight threads, the stress of the web comes out to be:

$$\sigma = \frac{T_1}{8(\pi(0.0025)^2)} = 1.247GPa. \tag{4}$$

Meanwhile, the strain is the ratio of the change in length to the original length of the web, which is given by:

$$\varepsilon = \frac{1}{y_1}(\frac{x}{cos\theta} - y_1) = 40\% \tag{5}$$

This results in a Young's modulus of $Y = \frac{\sigma}{\varepsilon} = 3.12GPa$.

Discussion

The calculated Young's modulus, while high, is very reasonable for that of spider's silk. The largest modulus is found in the silk of the orb-weaver family of spiders with Youngs moduli measured up to 12GPa [4], while other species have been known to have moduli of around 1.5GPa [5]. Interestingly, by taking the rest point as the point at which the silk is under its yield stress, that is when $\sigma \approx 1.3$GPa, we also obtain similar values to that of actual spider silk, falling somewhere between 1.1 and 1.5GPa. The toughness of this silk can also be calculated to be almost $500MJm^{-3}$, which is in line with values for the Darwin's Bark Spider, an orb-weaver with the strongest known webbing of any spider [6].

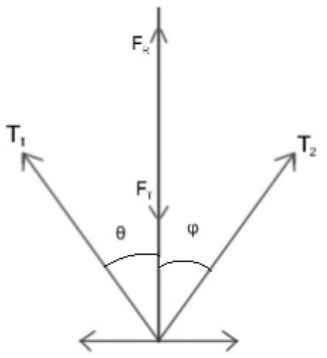

Fig. 1: Schematic Representation of forces acting on the train

Conclusion

Having determined these parameters, it can be stated that Spiderman's webbing is a proportional equivalent of that of a real spider, namely a weaker orb-weaver spider, but curiously, with a toughness more akin to some of the strongest spider silks.

References

[1] `http://www.youtube.com/watch?v=GYOYewO_Veg&noredirect=1`, a clip from Spiderman 2, accessed on 30/10/2012.

[2] `http://www.nycsubway.org/wiki/New_Technology_Trains_-_B_Division`, New York subway information, accessed on 30/10/2012.

[3] `http://www.engineeringtoolbox.com/rolling-friction-resistance-d_1303.html` accessed on 12/11/12

[4] M. Denny, *The Physical Properties of Spiders Silk and Their Role in the Design of Orb-Webs* Duke University, NC, 1976

[5] M. Bonino, *Material Properties of Spider Silk* University of Rochester, 2003

[6] I. Agnarsson, M. Kuntner, T.A. Blackledge, *Bioprospecting Finds the Toughest Biological Material: Extraordinary Silk from a Giant Riverine Orb Spider*, 2010