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P3_ 9 Solar Sails

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Abstract

The goal of this paper was to determine if it would be possible to leave the Solar System using the power of a solar sail alone. It was determined to be unrealistic as for a 1 kg mass, a square sail with sides of length 267 km would be needed to overcome the gravitational force due the Sun.

Introduction

This paper investigates if it would be possible to escape the Solar System using a solar sail. A solar sail works by using a photons momentum to create a radiation pressure which is exerted upon the sail.

For the purpose of this paper, it is assumed that the Sun and the solar sail are the only entities, i.e. there are no other bodies in the solar system. It is also assumed that the Sun acts as a point source for all calculations. This is a good approximation considering the vast distances between bodies in the solar system. The third assumption is that the sail perfectly reflects the incoming photons emitted from the Sun and the final assumption is that the initial velocity of the sail is zero but is position independent.

Gravitational Force

The first step is to find how the force on the sail due to the Sun's gravity varies with distance from the Sun. Equation 1 was used for this purpose,

$$F_g = \frac{GM_\odot m}{r^2}, \tag{1}$$

where F_g refers to the gravitational force on a body with mass m, a distance r away from the Sun of mass $M_\odot$ = 1.99x10^{30} kg [1], and G is the gravitational constant. It can be seen that the gravitational force drops as the distance between the bodies increases. Theoretically, if the sail started with no initial velocity, for the sail to accelerate out of the Solar System the radiation pressure would have to be greater than this gravitational force.

Radiation Pressure

The next step is to derive an expression describing how the force due to radiation pressure varies with respect to the distance from the Sun. This will then be compared with the gravitational force. Using equation 2 is a good starting point, it was found by multiplying the radiative pressure [2], by area to produce a force incident on the sail,

$$F_{rs} = \frac{A_\% a_s \beta_r T^4}{3}, \tag{2}$$

where F_{rs} is the force on the solar sail due to radiation pressure, a_s is the area of the solar sail, β_r is the radiation constant (7.57x10^{-16} Jm^{-3}K^{-4} [2]) and T is the temperature of the star propelling the sail. In this case 6000K [3] was used for the best case scenario as it represents the highest temperature regions of the Sun's photosphere. $A_\%$ refers to how much of the flux given off by the Sun would hit the sail, as the sail will experience less of a force at larger radii due to the inverse square drop off of radiation. An expression is given by taking the ratio of the area of the sail, a_s, to the area of the sphere the photons are emitted into at a distance r from the Sun,

$$A_\% = \frac{a_s}{4\pi r^2}. \tag{3}$$

Substituting equation (3) into (2) gives,

$$F_{rs} = \frac{a_s^2 \beta_r T^4}{12\pi r^2}, \tag{4}$$

which describes how the radiative force varies with distance from the Sun. The next step is to compare the force acting upon the solar sail.

Comparing Forces

Given that the initial velocity is zero, the radiative force needs to be greater than the gravitational force as this would allow the sail to escape the Solar System. It would accelerate away from the Sun overcoming its gravitational force due to the greater radiative force. These forces are orientated in opposite directions, hence for the sail to escape,

$$\frac{a_s^2 \beta_r T^4}{12\pi r^2} > \frac{GM_\odot m}{r^2}. \tag{5}$$

The radius terms can be cancelled from equation (5), then the only variables left are the area of the sail and its mass. Thus a relationship can be found to show how big the sail needs to be to carry a certain mass. This is given in equation (6),

$$a_s > 7.14 \times 10^{10} \sqrt{m}. \tag{6}$$

Using equation (6), it can be determined that a square sail with sides at least 267 km long would be needed for a 1 kg sail to escape the Solar System.

Conclusion

A colossal sail would be needed to escape the Solar System, leading to the conclusion that it would not be convenient to leave the Solar System using the power of a solar sail alone, although it is still possible. Problems would arise in deploying such a sail and manufacturing a sail of such size on Earth, making the idea unrealistic.

References

[1] *Sun Fact Sheet* (2004), NASA, cited 10/11/11,
http://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html
[2] *Stellar Radiation Pressure*, Hyperphysics, cited 10/11/11
http://hyperphysics.phy-astr.gsu.edu/hbase/starlog/staradpre.html
[3] *The Sun* (2010), NASA, cited 10/11/11
http://imagine.gsfc.nasa.gov/docs/science/know_l1/sun.html