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P1_8 Muonic Atoms

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Abstract

Muonic atoms are atoms where electrons are replaced with negatively charged muons. This paper analyses the implications of muonic hydrogenic atoms compared to standard hydrogenic atoms. It was found that energy levels in muonic hydrogen are increased by a factor of 186, whilst the Bohr radius is 186 times smaller. The spectra associated with Hydrogen are also significantly altered.

Introduction

The origin of the current accepted model of the hydrogen atom can be traced back to the work by Neils Bohr in 1912 [1]. The quantum mechanical description of the hydrogen atom develops Bohr's model, where electrons are in bound states described by a wavefunction that satisfies the Schrödinger equation for the hydrogen atom. This article explores what would be the effects if muons replaced electrons in their role in the hydrogen atom (and other atoms). Muons are leptons like electrons and only differ from electrons in one aspect: they are about 200 times more massive.

Quantum Mechanics of the Hydrogen Atom

To determine a solution for the energy levels in the hydrogen atom, it is necessary to solve the time-independent Schrödinger equation in three-dimensions. A detailed description of this is given by Rae [2], and the wavefunction is then applied to the hydrogen atom to derive the energy levels in hydrogenic atoms. These are derived by Rae [3], as:

$$E_n = -\frac{\mu Z^2 e^4}{32\pi^2 \varepsilon_0^2 \hbar^2 n^2}, \quad (1)$$

where Z is the proton number of the positive nucleus and e is the electronic charge. Equally ε_0 is the permittivity of free space, $\hbar$ is the reduced Planck constant and n is the quantum number defining the order of the energy level; the groundstate corresponds to a value of $n = 1$. A key parameter to note is μ, which is the reduced mass of an electron. The reduced mass of a particle, under the influence of a positive nucleus, accounts for the motion of the nucleus due to their mutual attraction. The reduced mass of any negatively charged particle p, subject to the potential of a positive nucleus N, is given by [3]:

$$\frac{m_N m_p}{m_N + m_p}, \quad (2)$$

where m_N and m_p are respectively, the mass of the nucleus and the particle.

Muonic Atoms

The quantum mechanics developed in the previous section can be implemented in exploring the effect of replacing electrons with muons. Throughout this article the mass of an electron and a muon are taken as $9.10938291\times 10^{-31}$ kg and $1.883531475\times 10^{-28}$ kg, and the mass of a proton (the positive nucleus) is taken as $1.672621777\times 10^{-27}$, since these are the current recommended values [4]. Substituting the appropriate values into Eq. 2 yields a reduced mass of a muon of 186 times larger than the reduced mass of an electron.

On substituting electrons for muons, the only parameter that changes in Eq. 1 is the reduced mass. Hence, the magnitude of the groundstate energy of a muonic hydrogen atom should be 186 times greater than that of a standard hydrogen atom. This is exactly the result that is obtained, the groundstate of a hydrogen atom has an associated energy of -2.18×10^{-18} J (-13.6 eV) where as for a muonic hydrogen atom the corresponding energy is -4.05×10^{-16} J (-2.53 keV). The fundamental constants in Eq. 1 are taken throughout this paper as the values provided by Rae [3].

It is also possible to analyse how the wavefunctions of a muonic hydrogen atom differ from the wavefunctions of the standard hydrogen atom. First, it is useful to define the Bohr radius a_0 [3]:

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}, \quad (3)$$

where the parameters retain their definitions from Eq. 1. Now, the radial wavefunctions can be analysed as a function of radius in units of a_0 as demonstrated by Rae [3]. The key result from analysing the probability distribution of the radial wavefunctions is that the Bohr radius defines the most probable position of the electron in the groundstate of a hydrogen atom, and then the most probable position of the electron in excited states of the atom is a radial position, greater than the Bohr radius.

The Bohr radius was calculated for a standard hydrogen atom and a muonic atom using Eq. 3, and it was found that for in a muonic atom a_0 is reduced by a factor of 5.38×10^{-3} (or 186 times smaller) when compared to a hydrogen atom. This result is very revealing as it portrays the most probable radial distance of muons at each energy state as being smaller than that of electrons in the same state, by a factor of 186. Effectively this means that muonic hydrogenic atoms can be considered as smaller than their electronic counterparts by a factor of 186.

Spectra

So far only the energy at discrete levels has been considered, but muon transitions between energy states will also produce different spectra. There are a number of series in emission and absorption spectra that can identify the presence of a particular atom, most notably the Lyman, Balmer, Paschen and Brackett series [5]. The frequency υ of a line in one of these series is given by the formula, developed by Rydberg in 1890, whose constant $\mathcal{R}$ appears in the formula [5].

$$\upsilon = -c\mathcal{R}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right). \quad (4)$$

In Eq. 4, c is the speed of light in a vacuum and the electron transition is from the state described by integer $n_1$to the state described by smaller integer n_2. Given that the Rydberg constant has the form [5]:

$$\mathcal{R} = \frac{\mu e^4}{8\varepsilon_0^2 h^3 c}.$$

It is clear that interchanging muons and electrons will simply increase the Rydberg constant. Consequently the entire series of absorption or emission lines described by Eq. 4 is expanded by a factor of 186. Aside from the fact that the spectrum would then no longer be able to identify atomic hydrogen, it would be shifted to much more energetic radiation. The Lyman series is usually towards the ultraviolet end of the visible spectrum [5] but would be shifted to well within the x-ray region, whilst the Balmer series, which is usually at the red end of the visible spectrum [5], would be shifted to the borderline between ultraviolet and x-ray radiation.

Conclusion

It is clear that muonic atoms have increased energy levels and decreased overall size, by a factor of 186, when compared to their electronic counterparts. Futhermore interchanging muons and electrons drastically changes the familiar absorption and emission spectra used to identify elemental hydrogen; conventional spectroscopic are no longer valid for detecting muonic atoms.

References

[1] P. Tipler and G. Mosca, “Physics for Scientists and Engineers, Sixth Edition”, W. H. Freeman and Co., New York, 2008. (pp. 1229)

[2] A. I. M. Rae, “Quantum Mechanics: 5th Edition”, Taylor & Francis, Boca Raton, FL, 2008. (pp. 48 - 57)

[3] A. I. M. Rae, “Quantum Mechanics: 5th Edition”, Taylor & Francis, Boca Raton, FL, 2008. (pp. 57 – 64)

[4] CODATA, “2010 CODATA Recommended Values: Atomic and Nuclear”, Committee on Data for Science and Technology (CODATA), CODATA Task Force on Fundamental Constants, 2011.
http://physics.nist.gov/cuu/Constants/
[Accessed: 14/11/11]

[5] T. P. Softley, “Atomic Spectra”, Oxford University Press, 1994. (p. 18)