Journal of Physics Special Topics

P2_3 A Relative on Olympus Mons.

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October 26, 2011

Abstract

This paper is an investigation into a variation of the Twin Paradox in relativity. In this scenario the first twin is living atop Olympus Mons, at a height of 25km above the surface level, whereas the second twin is living on the surface of Mars. The purpose of this investigation is to ascertain whether or not there would be a discernable difference in the ages of the twins due to relativistic differences over of 25 Earth years. Due to the presence of a gravitational field, it was necessary to include the effect of general relativity as well as special relativity. It was found that there the first twin would be approximately 72 seconds older than the second twin.

Introduction

In the theory of relativity there is a thought experiment known as the twin paradox. It is typically used to demonstrate the effects of special relativity on time in different frames of reference. This paper explores a variation of this thought experiment; by including the effects of general relativity. It should be noted that both the special and the general theory of relativity can have contributions to the effect of time dilation (providing both frames of reference are within a gravitational field). In this variation, the first twin is living atop Olympus Mons (the largest mountain on Mars – and the solar system) and the second twin is living on the surface of Mars (or the Martian equivalent of sea level – which has been assumed to be the radius of Mars). Over a time period of 25 (Earth) years it is thought that there should be a discrepancy in the time measured by the first twin (atop Olympus Mons) and the time measured by the second twin (on the surface).

Characteristic	**Value**
Mass/ x10^{23}Kg	6.4219
Sidereal rotation period/ sec.	88642.44
Radius of Mars/ km.	3397
Height of Olympus Mars/ km.	25

Table 1: A table of the required properties of Mars. All values [1] and height from [2].

Special Relativity

The time dilation caused by special relativity involves the velocity of an object relative to an assumed stationary frame of reference. In this instance, much like the calculations made for a GPS satellite, a rotational frame of reference has been used such that there is a difference in the velocities of both twins. This is given by *equation (1)[3]*; which shows the velocity v of the first twin (atop Olympus Mons) relative to the velocity of the second twin.

$$v = \frac{2\pi(R_{Mars}+H)}{T_{Mars}} - \frac{2\pi R_{Mars}}{T_{Mars}} \quad , \qquad (1)$$

where R_{Mars} is the radius of Mars, H is the height of Olympus Mons and T_{Mars} is the Sidereal rotation period of Mars. With respect to the values in *Table 1*, v was calculated as 1.772 ms^{-1}.

Due to the difference in velocity, there is a difference in the time experienced by both twins. This can be seen from *equation (2)[4]*,

$$\tau' = \tau \left(1 - \frac{v^2}{c^2}\right)^{-0.5} \quad , \qquad (2)$$

where τ is the time experienced in the stationary reference frame, τ' is the time experienced in the moving frame, v retains its earlier definition and c is the speed of light (299792458 ms^{-1}).

Therefore, over 25 years there would be a difference of 3.5×10^{-5} seconds.

General Relativity

In general relativity the effects on time are dictated by the position of the frame of reference within a gravitational field. In this case, the field belongs to the body of Mars. Using the rotational frame of reference the Schwarzschild metric (with Newtonian constraints) is shown in *equation (3)[5].*

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - dr^2 - r^2 d\theta^2 - r^2 sin\theta d\phi^2 \text{ , (3)}$$

where *ds* is the time experience when in a gravitational field, *dt* is the proper time, *G* is the gravitational constant, $6.67\text{x}10^{-11}\text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$, *M* is the mass of Mars, *r* is the radial coordinate of the event and θ and ϕ are the latitudinal and longitudinal coordinates of the event. The Schwarzschild metric was simplified by assuming dr^2 and $d\theta^2$ are equal to zero and that θ is equal to ½π. The resultant equation was as follows,

$$dt = \frac{ds}{\sqrt{\left(1 - \frac{2GM}{rc^2} - \frac{v^2}{c^2}\right)}}, \tag{4}$$

where *v* is the velocity within the field. Since the argument of the square root in *equation(4)* would produce an imaginary result, a binomial expansion was applied and *equation (4)* was rearranged for both frames of reference,

$$dt = ds_{top}\left(1 + \frac{GM}{r_s c^2} + \frac{\frac{1}{2}v_s^2}{c^2}\right) , \tag{5}$$

$$dt = ds_{surface}\left(1 + \frac{GM}{(r_s + H)c^2} + \frac{1}{2}\frac{v_t^2}{c^2}\right), \tag{6}$$

where the subscripts *s* and *t* denote the position (the surface of mars or on top of Olympus Mons respectively) of the argument and the other terms retain their previous definitions. From these, it is possible to derive a ratio of the time measured by the first twin, *equation (5)*, and the time measured by the second twin, *equation (6).*

$$\frac{ds_{surface}}{ds_{top}} = \left(1 + \frac{GMH}{(r_s)^2 c^2} + \frac{Hv_s^2}{r_s c^2}\right) \ . \tag{7}$$

This gives a ratio fractionally larger than 1 which when rearranged to find the difference between $ds_{surface}$ and ds_{top}, gave the time dilation per second. Hence, over 25 years the time dilation caused by general relativity was calculated to be -72 seconds.

Conclusion

Putting together the effects of both special and general relativity it is found that the twin atop Olympus Mons would be approximately 3 seconds older than the second twin. This signifies that the effects of general relativity on this scale are much larger than the effects of special relativity. In other words, because of the rate of rotation of Mars, the relative velocities of both twins are not particularly large. As a result, the effects of special relativity are smaller in magnitude. However, since the height of Olympus Mons is a fraction of the radius of Mars, there would be a significant difference in the strength of the gravitational field at these points. Thus the effects of General relativity have a much greater impact on the differences in time measured by the twins. Whilst 72 seconds is not enough time to see a discernable difference in the ages of the twins, it is still a significant and measurable temporal difference.

Reference

[1] *Mars Fact Sheets* (2010, November 17). Retrieved October 24, 2011, from National Aeronautics and Space Adminastration: http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.htm

[2] *Olympus Mons*. (n.d.). Retrieved October 24, 2011, from National Aeronautical and Space Administration: http://marsprogram.jpl.nasa.gov/gallery/atlas/olympus-mons.html

[3] and [4] P. A. Tipler, *Physics for Scientists and Engineers* (New York, 1999), 4th Ed. p. 125 and p. 1248

[5]*Gravity Control and Warp Drive for Space Travel*. (2005). Retrieved October 24, 2010, from Time Travel Research Center: http://www.zamandayolculuk.com/cetinbal/HTMLdosya2/GravityWarpDrive.htm.