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A2_6 Computational railgun dynamics & back EMF

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Abstract

Railguns are projectile firing devices that operate on the basic principles of electro-magnetism. In this paper we describe a novel method of simulating the dynamics of such a gun by solving a system of differential equations numerically. We then use this method to explore the effect of back EMF on railgun dynamics.

Introduction & Theory

A railgun is a projectile weapon consisting of two parallel rails connected to a power supply. A projectile is placed on these rails and the bridge that it creates completes the circuit. The moving charges in the projectile then interact with the magnetic field produced by the current in the rails, accelerating the projectile along the rails. This paper analyses the dynamics of the railgun problem and suggests a numerical approach to modelling the time evolution of a railgun system.

To begin we must derive the force that a railgun projectile is propelled by. The Biot-Savart law gives the magnetic field B at a distance r from a semi-infinite wire carrying current I as $B = \mu_0 I/4\pi r$ [1], where μ_0 is the permeability of free space. We make the semi-infinite wire approximation because at the ends of our rails they would - to a close projectile - essentially seem to extend to infinity. Clearly more accuracy can be obtained by assuming a finite wire, though this is beyond the scope of this paper.

In the case of two parallel wires separated by a distance d, the field at distance r from one wire is simply the sum of the fields for the 2 individual wires. At a point directly between our 2 rails, the field is hence:

$$B = \frac{\mu_0 I}{4\pi}\left(\frac{1}{r} + \frac{1}{(d-r)}\right). \qquad (1)$$

The average field along a line perpendicular to the 2 wires that connects them directly is therefore given by

$$B_{avg} = \frac{1}{D}\frac{\mu_0 I}{4\pi}\int_A^{d-A}\left(\frac{1}{r} + \frac{1}{(d-r)}\right)dr, \qquad (2)$$

$$B_{avg} = \frac{\mu_0 I}{2\pi D} ln\left(\frac{d-A}{A}\right), \qquad (3)$$

where A is the radius of the rail. Here we have defined $D = d - 2A$ as the distance between the wires - the width of the projectile. The use of this integral implies that the projectile is infinitely thin - which may not be a valid approximation to make if the projectile is of a similar thickness to the rails. The existence of the Lorentz force implies that a current carrying wire of length D in a magnetic field B_{avg} experiences a force $F_{avg} = B_{avg} I D$ [2]. By combining this result with equation 3 we find that the projectile experiences a mean force

$$F_{avg} = \frac{\mu_0 I^2}{2\pi} ln\left(\frac{d-A}{A}\right). \qquad (4)$$

Other forces on the railgun include friction and air resistance, though for the purposes of this paper we assume that they are negligible.

Of course, propulsion is not the only effect of electromagnetism on the railgun system. We also must consider the effects of back EMF - voltage induced by the motion of the projectile that acts to reduce the total voltage across the rails. Several simplifications have already been made to our railgun system and it is tempting to disregard back EMF in the same fashion. Hence by modelling the back EMF in our simulation we aim to decide whether or not it does have a great effect.

To begin considerations of back EMF it first must be noted that a general railgun design includes a capacitor such that a high voltage and hence high current can be obtained from a standard power source. Therefore, the voltage applied to the circuit changes with time. The voltage across the circuit may be expressed as $V = V_c - \mathcal{E}$ where V_c is the voltage across the plates of the capacitor and $\mathcal{E}$ is the back EMF of the circuit.

We must also consider that the resistance in the wire changes with time. To see this, consider the resistance of a length of wire of length l, cross-sectional area A and resistivity ρ - a quantity which is given by $R = \rho l/A$. The resistance of the entire railgun system is simply that of the projectile plus that of the rails. Hence at a position x along the rails the resistance of the railgun circuit - assuming both projectile and rails are of the same resistivity - is

$$R(x) = \rho(2x/(\pi A^2) + D/(\pi a^2)). \qquad (5)$$

Thus as the railgun accelerates along the rails the resistance increases. These two facts mean that over time we expect the current and hence field of the gun to drop off rapidly - something which greatly complicates our back EMF considerations.

Lenz's law states that a changing magnetic flux produces an EMF that attempts to cancel out the change in flux. The magnitude of this EMF is given by $\mathcal{E}(t) = \frac{d\Phi}{dt}$ where Φ is the magnetic flux [3]. We may write the flux through the current loop of our railgun (consisting of the two rails, the projectile and supplementary wiring at the ends of the rails) as $\Phi = BxD$. To differentiate this we must observe that - as stated above - the value of B is varying with time and hence we find - using the product rule - that

$$\mathcal{E}(t) = \dot{B}xD + B\dot{x}D. \tag{6}$$

$\dot{x}$ is simply the velocity of our projectile, but we must clearly calculate a value for $\dot{B}$. Differentiating equation 3 and using $I = V/R$, we find

$$\dot{B} = \frac{R\dot{V} - V\dot{R}}{R^2}\frac{\mu_0}{2\pi d}ln\left(\frac{d-A}{A}\right). \tag{7}$$

Since $V = V_c - \mathcal{E}$ it must be true that $\dot{V} = \dot{V}_c - \dot{\mathcal{E}}$. We hence substitute equation 7 into equation 6 and rearrange to find an differential equation describing the rate of change of $\mathcal{E}$;

$$\dot{\mathcal{E}} = \frac{R}{kx}\left(B\dot{x} - \frac{\mathcal{E}}{D}\right) + \frac{\dot{R}(\mathcal{E} - V_c)}{R} + \dot{V}_C. \tag{8}$$

Here we have defined $k = (\mu_0 ln((d-A)/A))/2\pi d$. This back EMF serves two purposes. Firstly it reduces the voltage across the rails and hence reduces the current and force in the gun. Secondly it reduces the effective plate voltage and hence discharge rate of the capacitor.

Given this issue aswell as that of time-varying resistance, it is prudent to begin our treatment of capacitor discharge from first principles. Considering both the back EMF and plate voltage,the current in the system at time t can be described by

$$I(t) = (V_c(t) - \mathcal{E})/R(t) = \frac{dQ}{dT}, \tag{9}$$

where Q represents charge. For a capacitor, the relation

$$Q(t) = CV_c(t), \tag{10}$$

where C is capacitance holds true. This can be rewritten as $Q = CI(t)R(t) - C\mathcal{E}(t)$ and then combined with 9 to give

$$\frac{dQ}{dT} = \frac{Q(t)}{CR(t)} - \frac{\mathcal{E}(t)}{R(t)}. \tag{11}$$

From 10, we find that $\dot{Q}(t) = C\dot{V}_c(t)$ and hence

$$\frac{dV_C}{dT} = \frac{V_c(t) - \mathcal{E}(t)}{CR(t)}. \tag{12}$$

We hence have a differential equation which we may integrate to study this discharge of a capacitor in a circuit of varying resistance.

Method

An assessment of the system of equations derived above shows that it is not trivial to solve them analtically as they all rely on each other in a complex fashion. We hence set up a standard 2nd order Runge-Kutta integrator to solve the problem numerically. This method was chosen for its simplicity and accuracy [4]. It requires that we have a differential equation for each evolved variable that we cannot calculate directly at time t. We can distill the railgun problem down to 4 such critical variables; $V_c, \mathcal{E}, x$ and $\dot{x}$. We already have derived such differential equations for V_c and $\mathcal{E}$ - equations 12 and 8 respectively. The time derivative of $\dot{x}$ is simply the force (equation 4) divided by the mass of the projectile and the time derivative of x is simply $\dot{x}$. Hence we have established a complete description of the evolution of the railgun system - negating losses.

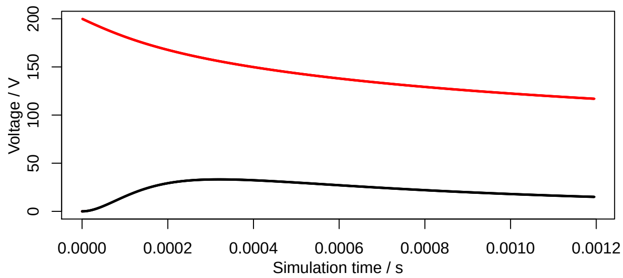

Fig. 1: Evolution of capacitor voltage (red) and back EMF (black) over time.

Discussion & Conclusion

An implementation of the above method was written in R and tested. To explore the properties of our method further, a plot was made of $\mathcal{E}$ and V_c over time (see figure 1). It is immediately clear that our assumption about back EMF is correct as the figure shows that it rises to a magnitude that is significant compared to the capacitor voltage. For contrast, we note that running the simulation from figure 1 without back EMF results in a 17.5% increase in final projectile velocity.

To conclude, we have shown that railgun dynamics can be approximated numerically. In doing so we have also demonstrated the importance of back EMF in a railgun system. In future work we will use our method to investigate the practicalities of a portable railgun system.

References

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