P3_10 Concrete Shoes

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Abstract

This paper aimed to explore the minimum additional weight required to successfully submerge a 70kg human body in the buoyant waters of the Dead Sea. The analysis showed that a human body could be sunk with an additional weight of 11kg or greater.

Introduction

The waters of the Dead Sea are famous for their extremely high salt content. The high salt content makes the waters more buoyant, thereby making objects float with ease. Humans in particular are able to float without any danger of sinking. This paper asked the question of how much additional weight it would take to actually manage to sink a human body in the buoyant waters.

Theory

In 212 B.C. Archimedes derived a theory of floating objects of which buoyancy is a part. Buoyancy refers to the situation where an object is being held at equilibrium on the surface of a fluid due to the difference in densities [1]. If the density of the object is lower than that of the fluid in which it resides then it displaces a volume of water and experiences a buoyant force equal to the weight of the water displaced.

Analysis

The weight of the body is given by the mass multiplied by the acceleration due to gravity. The mass was assumed to be 70kg. For the body to be in equilibrium, Archimedes principle becomes,

$$\frac{\rho_f}{\rho} = \frac{m_f}{m}, \qquad (1)$$

where ρ_f is the density of salt water, taken to be 1170kgm^{-3} [2], ρ is the density of the human body, taken to be 1010kgm^{-3} [3], m_f is the mass of the displaced water and m is the mass of the body. The force due to buoyancy is given by equation 2 [4],

$$F_b = \rho_f V g, \qquad (2)$$

where V is the displaced volume of the fluid and g is the acceleration due to gravity. Then equation (1) was rearranged terms of m_f / ρ_f and substituted into equation (2). The resultant was equated to the gravitational force.

$$mg = \frac{\rho_f}{\rho} mg \,, \qquad (3)$$

cancelling through by the mass and acceleration due to gravity on both sides of equation 3, shows the expected result. That is, when a body is fully submerged and is only just floating, the volume of the body will equal the volume of water displaced and, therefore, the densities are equal. Comparing the densities of the water and the body, it can be seen that in order to allow the body to sink the average density of the body must be increased by more than 160kgm^{-3}. The original mass and density of the body give a volume of 0.069m^{3}. With the new density of 1170kgm^{-3}, and the assumption that the change in volume will be negligible, the mass the body must gain is 11kg.

Conclusion

The analysis above shows that, despite the great buoyancy of the Dead Sea, a human body can indeed be made to sink with an additional mass of 11kg. This result is, however, dependant upon several assumptions; firstly that the mass added has negligible volume. This is assumed in order to allow the overall volume of the body to stay unchanged despite the mass input. Secondly, the body is assumed to be of average fat and muscle composition, as variances in this could alter the mass limit at which the buoyant force is overcome.

References

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