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A4_13 Stirling Engine

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Abstract

This paper examines the operation of Solar Stirling Engines by using the Sun's intensity as a power supply. It is found that the power output of a single array is significant and for the cost of an area of just over 800 acres the power output is comparable with a coal power station of 600 MW.

Introduction

Sterling Engines work similar to the internal combustion engine; except for one fundamental difference, instead of the heat source being internal, the piston chambers are heated from the outside. It is not a new concept; with the idea being conceived in 1816 [1] however they have not been put into common use due to the disadvantages of being slow to respond to changes. Figure 1 shows the working of a sterling engine, there are many different configurations yet they all work in the same way. The different configurations depend on the required use.

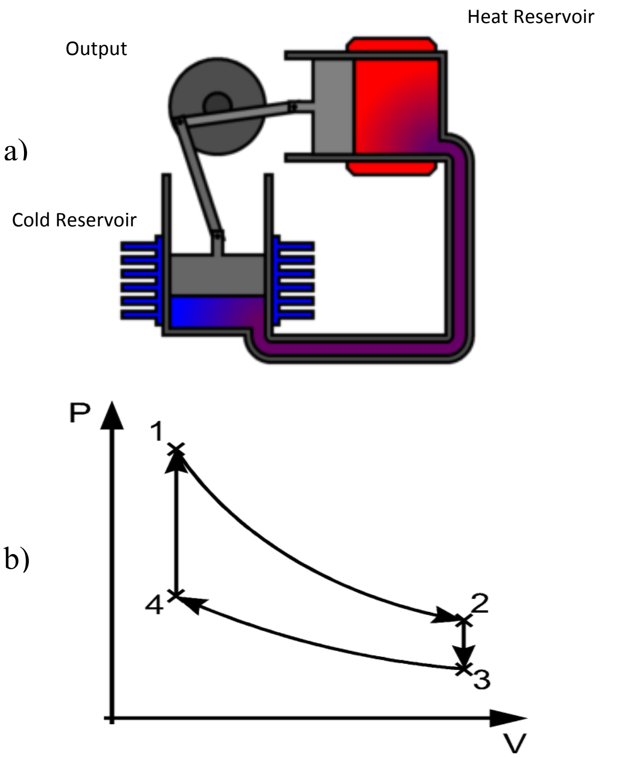

Figure 1: a) A diagram showing the configuration of an Alpha type Stirling Engine. (http://www.brighthub.com/engineering/mechanical/articles/3960.aspx, 30/11/11) b) Pressure-volume graph of the Stirling cycle.

This engine works in a four-stage process, seen in figure 1b:

1-2. Isothermal expansion, heat is taken from the hot source.
2-3. Adiabatic cooling
3-4. Isothermal compression, heat is removed by the cold source.
4-1. Adiabatic heating.

Temperature from the Sun

In order for the engine to work, one temperature reservoir has to be heated. One way to do this is by using the solar energy. It would be possible to use parabolic reflectors to heat a sterling engine and produce power, the entire assemble of reflector and engine is known as an array. The temperatures that these achieve, via this method, can be calculated by approximating the system as a blackbody. By assuming all the energy to heat the engine comes from focusing light off the reflector, which is assumed to be 100% efficient with the maximum energy being reflected off the entire area. To start of with the power incident on the reflector and then absorbed, P_{Abs}, [2] by the engine is;

$$P_{Abs} = (1-\alpha)P_S\left(\frac{\pi R_m^2}{4\pi D^2}\right). \quad (1)$$

Where α is the albedo, R_m is the radius of the mirror, P_s is the solar luminosity and D is the Sun-Earth distance. Next the power emitted, P_{emit}, by the engine is,

$$P_{emit} = \varepsilon\pi R_E^2\sigma T^4, \quad (2)$$

where ε is the emissivity, σ is the Stefan-Boltzmann constant, R_E is the radius of the engine and T is the temperature of the engine at the focus point. By equating these equations at equilibrium and rearranging for temperature then,

$$T^4 = (1-\alpha)P_S\left(\frac{R_m^2}{4D^2}\right)\frac{1}{\varepsilon\pi\sigma R_E^2}. \quad (3)$$

By using a typical radius of reflector and engine of 5.79 m and 0.18 m respectively [3] and assuming aluminium for the surface being reflected onto for its low emissivity of 0.09 [4] (although it has a high reflectance of 0.94, taken to be the albedo) a temperature of 2000 K is found.

Work out from the engine

In order to calculate how much work can be produced from the engine it is necessary to know how much work is done on, W_{on}, and by, W_{by}, the engine [5]. In the ideal case, the work, W, is the heat taken in from the hot source in an isothermal expansion, Q_H, minus the heat lost at the cold reservoir from isothermal compression, Q_C. Q_H and Q_C come from,

$$W = -\int_{V_i}^{V_f} PdV = -nRT\int_{V_i}^{V_f}\frac{dV}{V}, \quad (4)$$

when work is done by the system then there is no negative sign. So by looking at the Stirling cycle,

$$Q_H = W_{by} = nRT_H \ln\left(\frac{V_2}{V_1}\right), \quad (5)$$

$$Q_c = W_{on} = nRT_c \ln\left(\frac{V_3}{V_4}\right), \quad (6)$$

where n is the number of moles of fluid used (assumed 1) and T_H and T_c are the temperatures of the hot and cold reservoirs respectively. So by taking the work on the system from the work by the system and using the equation of quasi-static processes in an ideal gas, $TV^{\gamma-1}$= constant, then,

$$W = nR(T_H - T_c)\ln\left(\frac{V_2}{V_1}\right). \quad (7)$$

The only unknown value is for the compression ratio that will vary depending on the operation temperature and design of engine which is used. An average value of 5.54 was found [6] and thus used in equation 7 (cold reservoir assumed as 298 K), resulting in an output work of 24 kW.

The efficiency of this engine, e, is given by,

$$e = \frac{W}{Q_H}. \quad (8)$$

By using the values calculated, Q_H = 28.6 kW, thus far, an efficiency of 84% is found.

Discussion

The calculated power is higher than would actually be achieved due to several simplifications that have been made. The temperature would be lower than calculated due to the effects of heat conduction around the components of the engine and cooling effects by the wind, etc. The cold reservoir can be cooler than the temperature used due to the same effects.

In reality the efficiency of the engine would be significantly lower than calculated, from energy being lost in the system from places such as friction between the moving parts. However many papers have shown that Solar Stirling Engines are on the whole more efficient that standard solar arrays that can be produced [7].

The average coal power station produces 600 MW [8] corresponding to 25,000 Stirling arrays. Assuming no distance between the assemblies this is an area of 3350 thousand meters squared or 828 acres. This area is large compared to the size of an average coal power station but considering these arrays can be placed in areas where there is no civilisation, i.e. in deserts, then large amounts of clean energy can be produced.

Conclusion

Stirling Engines can provide a viable source of renewable energy; with continual improvements in the realms of material sciences then the efficiency of these devices will continue to approach the maximum theoretical efficiency. Further areas of investigations are within the internal working of the engine such as the fluid used inside and the materials used in construction.

References

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