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P4_14 "My God, It's Full of Star(s)"

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Abstract

A Dyson sphere is any hypothetical structure designed, by a very advanced civilisation, to capture all or almost all of the output of a star as useful energy. One simple configuration is a symmetric spherical shell, but this structure will not be kept in place by the star's gravity. This paper investigates if it can be kept in place by radiation pressure, but finds that without a system of active control to regulate how much momentum the solar radiation imparts to the sphere, it cannot be made stable through light pressure alone.

Introduction

The concept of a Dyson sphere was introduced in 1960 by physicist Freeman Dyson[1]. He hypothesised that an advanced civilisation in need of vast energy resources would construct a structure around its host star to capture all of that star's luminosity for their use.

There are multiple configurations of the Dyson sphere, but the one considered here is a simple, spherically symmetric shell of matter of uniform density.

Assumptions

The Dyson sphere under consideration is assumed to be constructed around a star identical to our own Sun in its present stage of life, at a distance of 1AU, and as in the original paper has a mass equal to that of Jupiter. The challenges of assembling such a structure have been extensively discussed elsewhere and will not be covered here. The shell is also assumed to be stable against gravitational collapse

Passive Stability

The process by which light pressure acts upon the shell is through the momentum of the photons it emits. Photon momentum is given by

$$p = \frac{E}{c} \quad (1)\ [2]$$

which does not contain a wavelength term, immediately eliminating a need to model the solar spectrum. The momentum imparted is calculated directly from the energy. For a star of constant luminosity, the energy seen by each unit area of the shell is simply a function of distance from the star.

For the case where the star lies at the exact centre of the shell, symmetry clearly shows no resultant force. If the star is offset from the centre of the shell by a distance x the situation is more complex. Distance from the star to an arbitrary point on the shell, r' is given in terms of the radius r by

$$r'^2 = x^2 + r^2 + 2rx\cos\theta \,. \quad (2)$$

The rate of energy received at a distance r' from the star is the total energy output of the star divided by the surface area of a sphere at that distance

$$\frac{dE}{dt} = \frac{L_{sun}}{4\pi r'^2} \,. \quad (3)$$

Combining with equation (1) and calculating the component of force in the direction of x as shown in figure 1 gives

$$F_x = \frac{dp}{dt}\cos\theta' = \frac{L_{sun}\cos\theta'}{4\pi c r'^2} \quad (4)$$

$$F_x = \frac{L_{sun}(x + r\cos\theta)}{4\pi c\left[x^2 + r^2 + 2rx\cos\theta\right]^{3/2}} \quad . \quad (5)$$

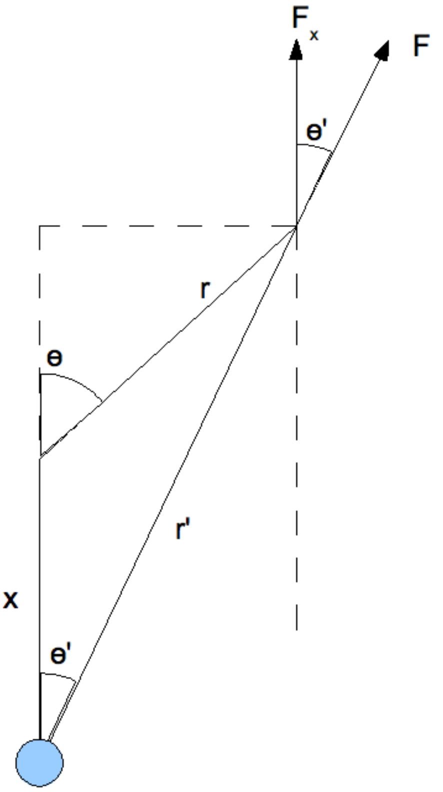

Figure 1 – force diagram for area element of shell

The integral of this force over the sphere is only dependent on one angle, so the other can be reduced to a constant, but is still complicated. An approximate solution near *x*=0 is given [3] by

$$F_x = \frac{L_{sun}}{2c}\left[\frac{\theta\sqrt{(x+r)^2}}{(x+r)^3}\right]_0^{\pi} . \quad (6)$$

As *x+r* is always positive, the force is positive and thus a restoring force (given the layout of the diagram). As *r* is much greater than *x* for small displacements, the magnitude of the force can be estimated, using a value for the luminosity of the Sun of $3.836\text{x}10^{26}$W, the speed of light of $3\text{x}10^{8}$m/s, and an Earth-Sun distance of $1.5\text{x}10^{11}$m[4]

$$F_x = \frac{L_{sun}}{2c}\frac{\pi}{r^2} = 8.93 \times 10^{-5} N \quad (7)$$

which, given the shell has a mass equal to that of Jupiter in the original paper, is not a large force.

Destabilising Impact

The work required to displace a shell-like Dyson sphere into its host star can be calculated by integrating the restoring force over the distance

$$W = \frac{\pi L_{sun}}{2c}\int_0^r \frac{1}{(x+r)^2}dx \quad (8)$$

which can be solved through the simple substitution A=x+r to give a value for W of $6.70\text{x}10^{6}$J

Given the escape velocity of our Sun at Earth's orbit is 32.74km/s [5] and taking this as a good estimate of the velocity of a random impact, the mass required to fatally destabilise a shell like sphere is

$$m = \frac{2E}{v^2} = 0.013kg \quad (9)$$

Conclusion

Although the light pressure of the host star does provide a restoring force to a shell like Dyson sphere, it is so small, on the scale of interplanetary impacts, that it will not provide any notable stability, and other Dyson sphere designs that are not gravitationally neutral should be considered instead.

References

[1] Dyson F., Search for Artificial Stellar Sources of Infrared Radiation, *Science* 1667

[2] Graham Woan, Cambridge Handbook of Physical Formulas pp. 90

[3] Wolfram Alpha calculation of indefinite integral (retrieved 15/03/11) http://www.wolframalpha.com/input/?fp=1&i=integrate+%28a%2bb+cos+x%29%2f%28%28a^2%2bb^2%2b2ab+cos+x%29^%283%2f2%29%29&_=1300274771299&s=26&incTime=true

[4] Graham Woan, Cambridge Handbook of Physical Formulas pp. 176

[5] Martin J. L. Turner "Expedition Mars" pp. 83