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A3_9 How fast can a pen write?

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Abstract

This article finds out the maximum speed in which someone could write on a paper using a ballpoint pen and how this is affected by temperature; where we take two different regions in the world as an example. It is found that this speed is around 153 m/s at room temperature.

Introduction

The first commercially significant ballpoint pen became available by the two inventors, László Biro and his brother Georg. Their pen reached the height of sales in 1944. Over the years many improvements have been made in the various parts of the pen such as the ink, the ball and the body [1].

A ballpoint pen works using gravity to direct ink towards the paper when held nib down. It consists of a tip that is automatically refreshed with ink. It has a metal ball which is seated in a socket below the reservoir of ink and exposed at the tip. As the tip of the pen is moved along a writing surface the small metal ball rotates and is constantly bathed by viscous ink from the reservoir.

The rate at which ink is delivered onto a writing surface such as paper and hence the rate at which a pen can write is dependent on the ink flow rate inside the inner tube of the pen's casing. Here we find out the maximum speed at which a ballpoint pen can write before no ink is delivered onto the paper. We then investigate how this speed varies in two different places around the world where there is a significant temperature difference between these regions.

Investigation

We consider the ink as an incompressible viscous fluid that flows through a horizontal tube of uniform circular cross-section. The pressuring gas at the far end of the inner tube allows a pressure difference to exist between the ends, for the ink to move down towards the tip. To calculate the volumetric flow rate Q, of the ink of viscosity η through a tube of length l and radius a, we use Poiseuille's equation given as [2]:

$$Q = \frac{\pi \Delta P}{8\eta l} a^4 \ , \qquad (1)$$

where ΔP is the pressure difference between the ends of the pipe. We consider a ballpoint pen with a typical inner tube length of 0.12 m and a radius of 2.5×10^{-3} m, containing an ink of viscosity 10-50 Pa·s [3] at room temperature (i.e. 25°). The absolute pressure of the pressuring gas ΔP is around 150 to 400 $\times 10^3$ Pa [2]. Taking the median of the range for the viscosity and the gas pressure (i.e. $\nu = 30$ Pa·s and $\Delta P = 225 \times 10^3$ Pa) we find the flow rate, Q, to be $9.59 \times 10^{-7}\ \mathrm{m^3 s^{-1}}$.

The speed at which ink is delivered onto the paper is dependent on the speed of a person's hand and is given by:

$$Q_p = sdv \ , \qquad (2)$$

where Q_p is the volumetric flow rate onto the paper, s and, d are the width and the thickness of the ink on the paper and v is the velocity of the person's hand as they write.

We now equate (1) and (2) to get an expression in terms of v to find the maximum speed at which a ballpoint pen can dispense ink onto a writing surface:

$$\frac{\pi\Delta P}{8\eta l}a^4 = sdv \,, \quad (3)$$

rearranging gives:

$$v = \frac{\pi\Delta P}{8\eta lsd}a^4 \,. \quad (4)$$

We assume s to be half the diameter of the inner tube of the pen (i.e. 1.25×10^{-3} m) and d to be 5 µm [4]. Substituting these values and the values for $\Delta P, \eta, a$ and l used above into (4) give v to be 153 m/s.

We now wish to find how the maximum speed a ballpoint pen can write varies across two different regions in the world by picking two extremes; the temperature of the Sahara desert in summer (~323 K [5]) and that of Siberia during winter (~243 K [6]). Viscosity η varies with temperature T, generally becoming smaller as temperature is elevated and we can use Arrehenius's model [7] to calculate this temperature dependence on viscosity as:

$$\eta = \eta_o \exp\left({}^{E}/_{RT}\right) , \quad (5)$$

where η_o is a reference viscosity in Pa· s, E is the activation energy of the material in J/kg, R is the universal gas constant (8.31 J/mol.K [8]) and T is the temperature in K. The viscosity profile of ink is very similar to honey and we can take the activation energy of honey to be equal to that of ink, given as $92.3\times 10^3 \text{ J mol}^{-1}$ [9], which is 449 Jkg^{-1}. Taking η_o to be 30 Pa· s as the viscosity used earlier for ink at room temperature, we find η for Sahara desert and Siberia with respect to η_o as 25.4 and 24.0 Pa· s. Substituting these values into (4) and keeping all the other variables the same as before, we get v in Sahara desert and Siberia to be 181 m/s and 192 m/s respectively.

Discussion

Even though our maximum speed calculated for a pen to write in room temperature and both extreme conditions considered are in practice not feasible, this investigation has shown that there is a considerable difference in flow rates of ink onto a writing surface for these different regions but as long as v is maintained low, the pen would write just as well in any place around the world.

When using (5) to calculate the values for viscosity we used a simple model. In reality the temperature dependence on viscosity is more complicated with many factors involved such as concentration of solution, shear rate, ionic strength etc. There is no one generalized model for all types of materials.

We also assumed in our investigation that the ink is slow drying in the reservoir, and free of particles. These characteristics ensure that the ink continues to flow to the paper without clogging the ball. As a matter of fact gaps are eventually created in the inner tube on the ballpoint pen and ink delivered onto the paper will stop for a period of time until these gaps are filled again. This happens because of friction between the ink and the walls of the inner tube. These gaps are the likely reason why ink flow from ballpoint pens onto a paper stops when one writes at a modestly fast pace.

References

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