P4_11 Solar Wind

F. Tilley, C. Davis, P. Hague

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

March 2, 2011

Abstract
The physics behind the solar wind is investigated in order to determine whether it is possible to have a star whose solar pressure could be held back by the local interstellar medium, thus placing it in hydrostatic equilibrium with no solar wind. It is found using a simplified model that such a star probably cannot exist.

Introduction
Every second the Sun streams out $10^9$ kg [1] of matter in the form of the solar wind. This outflow of mass is caused by an imbalance between the gravitational forces and the pressure caused by the intense heat generated at the Sun’s core. One way of halting this and putting the star in hydrostatic equilibrium would be if the total outflow pressure was less than or equal to the pressure of the local interstellar medium (LISM). This paper investigates the possibility of having a star in hydrostatic equilibrium with the LISM.

Theory
To calculate the pressure, $P$, of a plasma with energy $kT$, we can use a modified version of the momentum equation [1]

$$\frac{dP}{dr} = -\frac{\langle m \rangle g}{kT} P,$$

where $\langle m \rangle$ is the average particle mass and $g$ is acceleration due to gravity. When used to model a thick atmosphere like the one around a star, this can be integrated to get the pressure at a distance $r$

$$P(r) = (2nkT) \exp \left[ -\frac{GM(m)}{kT} \left( \frac{1}{r} - \frac{1}{R} \right) \right].$$

Here $n$ is the particle density, $M$ and $R$ are the mass and radius of the star and $G$ is the gravitational constant. In order to solve this, a number of assumptions and simplifications will have to be used.

<table>
<thead>
<tr>
<th>Mass ($M/M_\odot$)</th>
<th>Radius ($R/R_\odot$)</th>
<th>Surface Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.20</td>
<td>2.50</td>
<td>10800</td>
</tr>
<tr>
<td>2.10</td>
<td>1.70</td>
<td>8620</td>
</tr>
<tr>
<td>1.70</td>
<td>1.40</td>
<td>7240</td>
</tr>
<tr>
<td>1.29</td>
<td>1.20</td>
<td>6540</td>
</tr>
<tr>
<td>1.10</td>
<td>1.05</td>
<td>6000</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>5920</td>
</tr>
<tr>
<td>0.93</td>
<td>0.93</td>
<td>5610</td>
</tr>
<tr>
<td>0.78</td>
<td>0.85</td>
<td>5150</td>
</tr>
<tr>
<td>0.69</td>
<td>0.74</td>
<td>4640</td>
</tr>
<tr>
<td>0.47</td>
<td>0.63</td>
<td>3920</td>
</tr>
<tr>
<td>0.21</td>
<td>0.32</td>
<td>3120</td>
</tr>
</tbody>
</table>

Table 1. Mass, radius and surface temperature values for 11 main-sequence stars [2].

Figure 1. A graph showing the relationship between mass and temperature (K) (blue) and mass and radius (m) (red) for the data in Tab. 1.
Firstly we will assume that hydrostatic equilibrium will occur at large \( r \) i.e. when \( P \) is lowest. Secondly we will assume a linear relationship between the mass of a star and its radius and surface temperature. This second assumption is backed up by the data shown in Tab. 1 and the graphs shown in Fig. 1. From this data we can estimate the relationships between mass, radius and temperature as

\[
T_{\text{surface}} = 1 \times 10^{-27} M + 3029.8, \quad (3)
\]
\[
R = 2 \times 10^{-22} M + 2 \times 10^8. \quad (4)
\]

A major issue arises with the temperature values. The temperature-mass relation only gives the surface temperature, and to work out the solar pressure we need the coronal temperature. The nature of coronal heating is still unknown and so in order to solve the equation we will have to make another assumption. We will assume that the temperature of the corona is linearly linked to the surface temperature, however we cannot test or investigate the validity of this assumption.

Using the values of the coronal [1] and surface temperatures [2] of the Sun we can estimate a multiplying factor to apply to all the other stars’ surface temperatures to get a link between mass and coronal temperature which comes out as

\[
T_{\text{corona}} = 4 \times 10^{-25} M + 1 \times 10^6. \quad (5)
\]

We can now apply the large \( r \) limit to Eqn. 2 and substitute in the relationships found in Eqns. 4 and 5 to get an expression for the pressure of a star’s atmosphere in terms of its mass.

\[
\ln(P) = \frac{1.14 \times 10^{25} \left( -2.37 \times 10^8 + 3.12 \times 10^6 \ln(M + 2.5 \times 10^{30}) \right)}{(M + 1 \times 10^{30})(M + 2.5 \times 10^{30})}, \quad (6)
\]

\( \ln(P) \) is used in order to simplify the expression. The pressure of the LISM is found to be \( \sim 10^{-13} \text{ N m}^{-2} \) [1] and we can plot the natural logarithm of this along with Eqn. 6 to see if there is a mass for which the solar pressure could be held back by the LISM. This can be seen in Fig. 2.

**Figure 2.** The plot of Eqn. 6 is shown as the blue line and the limit of the LISM is shown as the red line. At no value of \( M \) does the solar pressure become low enough to be halted by the LISM.

**Conclusion**

As Fig. 2 shows, this model predicts that no value of \( M \) will give a pressure low enough to be halted by the LISM at large \( r \) and thus all stars will have a solar wind of outflowing gas. The graph predicts a minimum pressure, and solar wind, for a star with a mass of around \( 3 \times 10^{30} \text{ kg} \).

Unfortunately, due to the nature of the assumptions made, especially the link between coronal temperature and solar mass, this cannot be used as a proof of the inevitability of solar winds. Hopefully with more research into solar coronae the physics behind this model can be refined.

**References**