# P1_2 Unmanned Aerial Vehicles 

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#### Abstract

In this article, we estimate the energy required for a sustained flight of 12 hours for a UAV (modelled on the in-development BAE Taranis) from London, United Kingdom to Baghdad, Iraq, and the amount of fuel needed for the journey. Approximations are made as to the physical parameters of the UAV, taking those known for similar-sized aircraft. We find that the energy consumption would be 30 GJ , equating to approximately 700 kg of aviation fuel.


## Introduction

When operating a surveillance mission using unmanned aerial vehicle (UAV), we may be interested the amount of energy required to complete a long-distance flight of the UAV. This paper will discuss aerodynamic and energy considerations of the flight, and the resultant fuel consumption of the craft.

## The UAV/Aircraft Model

We consider an unmanned combat aircraft vehicle (UCAV), with size parameters modelled on those of the BAE Taranis project, in development as of February 2011. The physical properties of the craft are listed by BAE [1]. The cross-sectional area of the UAV is difficult to calculate exactly, and so we model it as a central elliptical area, with two isosceles triangles representing the wings, based on the dimensions listed above. Combining the areas of the three geometric shapes comprising the model, we find that the cross-sectional area $(\sigma)$ is equal to $19.6 \mathrm{~m}^{2}$.


Figure 1: Model of the UAV cross section (front view).
The area of its underside (effective wing area) can be estimated by treating the shape of the craft as a simple triangle. We find this to be valid by comparison with the design of the Taranis, shown here relative to the size of a pilot [1].


Figure 2: Shape and relative scale of the UAV (top view)
The surface area can be approximated as that of a triangle, half the product of base and height (in this case width and length: 9.4 mx 11.35 m respectively): $53.345 \mathrm{~m}^{2}$.

## Calculations

Now that we have set up the geometry of our aircraft, we can consider the lift force necessary to keep the craft in the air at our chosen altitude. We take the steady-flight altitude to be $30,000 \mathrm{ft}$ above sea level (approximately 9 km ), based on the assumption that the Taranis - designed to 'inform decisions about a future long-range offensive aircraft and evaluating UAVs' - is a Medium Altitude-Long Endurance (MALE) craft, a classification of UAVs characterised by an altitude of $30,000 \mathrm{ft}$ and range greater than 200km[7].

We can say that the lift UAV experiences, at height, $h$, is [6]

$$
\begin{equation*}
F_{L}=\frac{A}{2} \rho(h) v^{2}\left(m^{2}-1\right)=M g \tag{1}
\end{equation*}
$$

where $A$ is the wing area, $\rho(h)$ is the density of air at height, $h ; \boldsymbol{v}$ is the relative velocity of the craft and $m$ is the ratio of path lengths above and below the UAV wing. We take this to be 1.1 for the purposes of this model; a $10 \%$
longer path above the wing. This is an estimate, corresponding to an angle of attack of the aerofoil of $9^{\circ}$, for the similarly-sized Cessna 172 based on thin aerofoil theory [8].

When this is equal to the weight, $M g$, of the UAV (assuming there is no appreciable mass loss through fuel burn, i.e. mass remains constant), we can sustain flight at altitude, $h$. Given that the mass of the Taranis, $M$, is 8,000 $\mathrm{kg}, \boldsymbol{g}$ is $9.81 \mathrm{~ms}^{-2}$ and the density of air in the atmosphere falls exponentially, following the relationship [2]

$$
\begin{equation*}
\rho(h)=1.21 e^{-\frac{h}{8000}} \tag{2}
\end{equation*}
$$

This evaluates to $0.39+0.01 \mathrm{~kg} / \mathrm{m}^{3}$ for 9 km altitude. After some rearranging of (1), we can conclude that the necessary velocity to sustain flight at 9 km is

$$
\begin{equation*}
v^{2}=\frac{2 M g}{A \rho\left(m^{2}-1\right)} \tag{3}
\end{equation*}
$$

We need not assign a numerical value to this velocity yet, although it is wise to check at this stage that we do not have a nonsensical scenario, and so $v=189.5 \mathrm{~ms}^{-1}=423.9 \mathrm{mph}$. A similar aircraft, the BAE Hawk is capable of Mach 0.84 [1], or 639 mph in level flight, and so the necessary Taranis speed is reasonable.

If we wish to estimate the thrust necessary to provide this velocity, and to maintain the desired height, we can consider the situation whereby thrust and aerodynamic drag have equalised (equilibrium), or steady-flight. The expression for drag is given as [6]

$$
\begin{equation*}
D=\frac{1}{2} \rho \sigma C_{d} v^{2} \tag{4}
\end{equation*}
$$

where $C_{d}$ is the drag coefficient, taken to be 0.027 (that of the light aircraft Cessna 310, of similar dimensions[3]) and $\sigma$ is the crosssectional area, calculated previously to be $19.6 \mathrm{~m}^{2}$. Equating this to the necessary thrust, $T$, replacing $v^{2}$ with equation (3), and cancelling like-terms, we see that the thrust necessary to sustain flight at this altitude is

$$
\begin{equation*}
T=\frac{M g \sigma C_{d}}{A\left(m^{2}-1\right)} \tag{5}
\end{equation*}
$$

Substituting in our estimated and calculated values for the parameters yields a necessary thrust of $\boldsymbol{T}=3.707 \pm 0.05 \mathrm{kN}$. This thrust is well within the capability of the Taranis [1]. Given that power, $P=\boldsymbol{F} . \boldsymbol{v}$ [6], we can say that the power of the UAV during steady flight is $702 \pm 1 \mathrm{~kW}$.

## Discussion

We may wish to consider a particular example: a flight from London, United Kingdom to Baghdad, Iraq. The direct, roundtrip distance between these two cities is 8172.64 km [4]. At the velocity calculated using equation (3), the Taranis would complete this journey in approximately 11.98 hours. During this time, we can calculate the energy used by the UAV, through the simple relation $E=\int_{0}^{t} P . d t[6]$

## Conclusion

The flight total energy evaluates to $30 \pm 0.5$ GJ for the duration of the flight. Assuming that the Taranis is powered by combustion of traditional aviation fuel, of average energy density $43 \mathrm{MJ} / \mathrm{kg}$ [5], we estimate the mass of fuel necessary for the round-trip journey to Baghdad to be approximately 698 kg . This is less than $10 \%$ of the mass of the UAV, and so we suggest that this constitutes a reasonable prediction for the fuel consumption of the Taranis as modelled in this article.

## References

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