A3_4 Playing Pool

Karazhov D., Ryan L., Booth T.

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

February 28, 2011

Abstract
In this article we investigate whether large angle shots in cue ball games, like Pool or Snooker, have an effect on accuracy. It was concluded that 0° to 40° shots introduce smallest uncertainty, and that shots over 70° have asymptotically large uncertainty.

Introduction
Pool and snooker have gained huge popularity around the world and are some of the most played cue ball games today [1].

The successfulness of a player depends on his accuracy, i.e. uncertainty in the angle. There are many other factors that matter, e.g. distances between balls, pocket sizes etc.

In this article we shall concentrate our attention on dependency of the accuracy on the angle of attack, \( \phi \).

Figure 1: Diagram showing an example when there are two possibilities of potting a ball into two different pockets with different angles \( \phi \).

Figure 1 shows an example in a pool or snooker game when the angle of attack factor plays an important role in influencing player’s choice. Our goal is to find out which shot should be taken for increased probability of success.

Model
We consider the cue ball a distance \( d \) from the target ball, as an initial configuration outlined in figure 2.

In order to simplify calculus, we may safely consider the cue ball as a point particle. The target ball should then have radius \( R = R_{\text{cue}} + R_{\text{main}} \).

Geometrical diagram

\[ \begin{align*}
  \text{Figure 2: Geometrical 2-D configuration of the system. The cue ball is situated at } (x, y) &= (-d, 0) \text{ with radius } \rightarrow 0, \text{ whereas the target ball is at } (0, 0) \text{ with radius } R. \\
\end{align*} \]

The angle of attack \( \phi \) is a well-defined function of \( \gamma \). The (fixed) uncertainty in angle \( \gamma \) (within an agreed probability percentage) defines “how good a player is” and this value does not depend on angle \( \phi \). To find how probability of success depends on \( \phi \) we need to find \( \frac{d\phi}{d\gamma} \) as a function of \( \phi \). This quantity...
translates fixed uncertainty in $\gamma$ into probability of success as a function of angle of attack $\varphi$.

We solve these equations numerically.

**Derivations and results**

The path of the cue ball prior to collision can be expressed as a linear equation with gradient $\tan(\gamma)$:

$$y = \tan(\gamma)(x + d).$$  \hspace{1cm} (1)

The target ball has a general equation of circle:

$$x^2 + y^2 = R^2.$$  \hspace{1cm} (2)

For the collision point $(x_c, y_c)$ we equate $x$ and $y$ of both equations. Upon substitution of equation (1) into (2) and solving for $x_c$ (only considering negative value) we arrive at:

$$x_c = -\left(\frac{d\tan^2(\gamma)+\sqrt{d^2\tan^4(\gamma)-(\tan^2(\gamma)+1)(d^2\tan^2(\gamma)-R^2)}}{\tan^2(\gamma)+1}\right).$$  \hspace{1cm} (3)

In order to relate $x_c$ to $\varphi$ we notice, with an aid of figure 2, that:

$$-x_c = R\cos(\varphi).$$  \hspace{1cm} (4)

Now $\varphi(\gamma)$ is easily found from (3) and (4), likewise $\frac{d\varphi}{d\gamma}$ as a function of $\gamma$.

The derivative is a function of $\gamma$, but we require it to be function of $\varphi$ for appropriate analysis.

Unfortunately, $\varphi(\gamma)$, like many mathematical functions, cannot be inversed to find $\gamma(\varphi)$ and substituted into $\frac{d\varphi}{d\gamma}$, so numerical values are extracted from the function at equal intervals of $\gamma$ and equated to $\varphi(\gamma)$. The results are shown in figure 4 for typical values of $d$ and $R$ (1 m and 0.1 m respectively).

From the figure it is obvious that the two factors have large dependency on each other. The accuracy stays approximately constant between 0 and 40 degrees of $\varphi$. However, above 60° the uncertainty dispersion rises dramatically and this simply reduces chances of potting the ball at such angles.

**Conclusion**

Using analytical derivations and numerical extractions, it was shown that the probability of success of a pool or snooker player, when taking a shot, can dramatically depend on the angle of attack.

Shots at 0 to 40 degrees keep the accuracy approximately constant and at its best possible value, but at 70° it decreases by a factor of 5.

To conclude, shots at 60° or more should be avoided in order to attain better results.

**Bibliography:**