Feeling Lucky?

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Abstract

When playing the popular casino game Blackjack, the odds of getting a “blackjack” are quite low. The online gambling industry is growing rapidly, so when playing Blackjack online and the computer manages to win by getting multiple blackjacks in a row, was it due to “luck” or was the computer program cheating?

Introduction

Blackjack is a casino card game played with one or more standard decks of 52 playing cards. The basic outline of the game, without going into detail, is as follows. The player is dealt an initial two cards from the shuffled deck(s) and can then take additional cards, one at a time, to try and obtain a card total as close to 21 as possible without exceeding. Cards drawn are not replaced back into the deck, so after the first round, the game repeats with the remaining cards. The number of decks used can vary from a single deck, up to a maximum of 8 decks shuffled together and this can have an effect on the probability.

Probability

Obviously there is a greater chance of seeing blackjacks if half the cards have been drawn without any aces, 10s or face cards appearing. But we will consider the probability of obtaining multiple consecutive blackjacks at the start before any cards are drawn. Thus, the first two cards of the well shuffled deck(s) must form a blackjack. For the case in which only a single deck of cards is used, the probability of this is given by,

$$P_1(1) = 2 \times \left( \frac{4}{52} \times \frac{16}{52-1} \right).$$

This is simply the probability of the first card being an ace and the second card being a ten or a face card, plus the probability of the first being a ten or face card then followed by an ace. Hence, when $n$ number of decks is used, the probability is given by,

$$P_1(n) = 2 \times \left( \frac{4n}{52n} \times \frac{16n}{52n-1} \right).$$

The probability that, on the $m$th round of cards drawn from a set of cards containing $n$ decks, is the $m$th consecutive blackjack in a row is given by,

$$P_m(n) = 2 \times \left( \frac{4n-(m-1)}{52n-(2m-2)} \times \frac{16n-(m-1)}{52n-(2m-1)} \right).$$

Discussion

As shown in Table 1, the probability of a blackjack in the first two of cards decreases with more decks, but the subsequent probabilities of a blackjack increases with more decks. The total probability of getting consecutive blackjacks is calculated by multiplying the individual probabilities of the number of consecutive blackjacks given that the previous draw of cards resulted in blackjacks. For example, the actual chance of getting two blackjacks at the start of playing is...
equal to the probability of a blackjack from the first draw, multiplied by the probability of a blackjack from the second draw given that the first draw was a blackjack. These are listed on the right hand side of Table 1 and shows that the probability increases with the number of decks used. It is very rare to see consecutive blackjacks no matter how many decks are used. You can expect to see two blackjacks in a row about 0.2% of the time. That is once in every 500 sets of two draws.

Fig. 1 shows that if a blackjack was drawn, then the chance was getting the next blackjack drops significantly the fewer number of decks you use. This is because, for example after the first blackjack is seen with a single deck there will only be 75% of aces left, but for 5 decks there will be 95% of aces left, so the loss of the cards that make up a blackjack becomes less important. The graph also flattens out as you increase the number of decks so its effect becomes insignificant in high numbers.

**Conclusion**

Clearly the likelihood of encountering consecutive blackjacks is dependent on the number of games you play. As with all games of chance, it is hard to differentiate between cheating and “luck” if the level of cheating employed is very subtle. A large sample of games will need to be played against the computer program that is under scrutiny and the results can then be compared to those in Table 1 to give an insight into the accuracy of the program. Then the standard deviation of the probability distribution can be analysed and a chosen confidence interval can be used to determine whether the program is likely to be rigged. The results of this paper should give a good base for further study on different Blackjack programs.

**References**


**Appendix**

Table 1. Calculated probabilities for values of $n$ from 1 to 8.

<table>
<thead>
<tr>
<th>Number of Decks</th>
<th>Chance of Blackjack From First Two Cards (%)</th>
<th>Chance of 2nd Consecutive Blackjack Given First (%)</th>
<th>Chance of 3rd Consecutive Blackjack Given Second (%)</th>
<th>Chance of 4th Consecutive Blackjack Given Third (%)</th>
<th>Chance of 5th Consecutive Blackjack Given Fourth (%)</th>
<th>Total Cumulative Probability (%)</th>
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<tbody>
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<td>1</td>
<td>4.83</td>
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<td>0.177</td>
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<td>2.46</td>
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<td>4.01</td>
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<td>0.219</td>
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</table>

Figure 1. A graph showing the probability of a next consecutive blackjack given previous blackjack vs. the number of decks.