Journal of Special Topics

P2_01 Gravitational Influence on the Pole Vault

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February 1st, 2011

Abstract

A recent popular web comic [1] claimed that differences in the acceleration due to gravity across the Earth could cause a significant difference in the heights attained by identical vaults in the pole vault. This paper seeks to quantify such variation and concludes that it can be as much as 1%.

Introduction

The Earth is a slight oblate spheroid; it is flatter at the poles and bulges at the equator. Thus acceleration due to gravity is greater at the poles, as they are closer to the centre of the Earth, and less strong at the equators. Centripetal force from the Earth's rotation is also important at the equator in reducing the effective gravity.

Major athletic competitions are held all over the world and whilst conditions are generally equal for those competing at the event, circumstances can change at different locations, which could offer an advantage in setting records. In the pole vault event athletes use a 3-5m pole to vault over a bar, the height of which increases to establish who can vault the highest. As the athletes are competing against gravity, the "same jump" could potentially result in them attaining a different height at different locations.

To investigate this the paper will compare the effective acceleration at the geographical pole and the equator. Ignoring all other variables the height of the pole vault should be inversely proportional to this acceleration.

Model

In our model the most significant variations in gravitational force are due to the shape of the Earth and centripetal force. The maximum possible difference is from the poles to the equator. Whilst no international event is likely to take place at the poles, this will serve as a useful limit. Topography and the density of the internal structure of the Earth do also play a part, however their effects are negligible in comparison.

As there is no centripetal force at the geographical poles the acceleration is purely gravitational, calculated using

$$g_p = \frac{GM}{{r_p}^2}\,, \quad (1)$$

where g_p is acceleration due to gravity at the poles, G (6.67300×10^{-11} m^3 kg^{-1} s^{-2}) [2] is the gravitational constant, M is the mass of the Earth (5.9742×10^{24} kilograms) [2] and r_p is the polar radius (6356.750 km) [3].

Centripetal force does have an effect at the equator and therefore the effective gravity is calculated using

$$g_{eq} = \frac{GM}{{r_{eq}}^2} - \frac{v^2}{r_{eq}}, \quad (2)$$

where g_{eq} is the effective acceleration at the equator, r_{eq} is the equatorial radius (6378.135 km) [3] and v is the rotational velocity of the point of the Earth's surface (464.486 ms^{-1} from equatorial circumference divided by the number of seconds in a day).

This gives the following. $g_p = 9.866ms^{-1}$ and $g_{eq} = 9.766ms^{-1}$, a percentage difference of $\sim 1\%$.

Neglecting air resistance the vault can be modelled using the standard SUVAT equations for movement with constant acceleration [2]

$$x = ut + \frac{1}{2}at^2 \text{ and } v = u + at.$$

Taking only the vertical component of the athlete's motion and assuming a constant starting velocity (u), downwards acceleration due to gravity (g) and velocity $v = 0$ at the top of the vault the two equations can be combined to yield

$$x = \frac{3u^2}{2g}. \quad (3)$$

From this it can be seen that the height of the vault, x, is directly, inversely proportional to the gravitational acceleration, g, i.e. neglecting all other variables a $\sim 1\%$ change in g will lead to a $\sim 1\%$ change in vault height.

Discussion and implications

A difference of 1% is potentially important in tightly contested events, and could significantly influence the final results. For example a world class vault for men is around 6m [4], on which 6cm is not negligible.

At the 2008 Beijing Olympic Games the difference between 1st and last (11th) place in the men's pole vault event was 51cm [5]. A 6cm difference is $\sim 12\%$ of this, further demonstrating that the effective gravity is an important factor.

However a complete model would need to take several other factors into account. Higher altitudes could possibly affect the body's ability to respire and provide energy; however it is likely this would be more important in endurance events. Conversely the higher altitudes thinner air would also lead to reduced air resistance, possibly influencing vault height. For example many world records were set in anaerobic events during the 1968 Olympic Games, which were held in Mexico City at an altitude of 2200m. The thinner air is believed to have been a key advantage to the competitors [6]. Temperature and weather differences could also be a factor.

Conclusion

In conclusion simple calculations demonstrate that variations in the acceleration due to gravity across the Earth's surface are potentially significant with regards to pole vaulting. These variations (caused by the shape of the Earth and centripetal acceleration) lead to a 1% difference in acceleration from the poles to the equator, which is directly proportional to the height of the vault. Other factors may also contribute such as air density and weather, and further work could be carried out to quantify these.

References

[1] http://xkcd.com/852/ (03/01/2011).
[2] P.A. Tipler and G. Mosca, *Physics for scientists and engineers*, 5th edition (2003).
[3] A. Cazenave, *"Geoid, Topography and Distribution of Landforms"* (1995).
[4] http://www.polevaultpower.com/6mclub.php (03/01/2011).
[5] http://www.abc.net.au/olympics/2008/results/at/mens-athletics-pole-vault.htm?result=s26987 (03/01/2011).
[6] M. N. Brearley, "*The Long Jump Miracle of Mexico City*" Mathematics Magazine, Vol. 45, No. 5. (1972).