An Introduction to Climate Modelling and the Impact of Climate Change on Average Temperature in Leicester

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Abstract

This paper will introduce a simple climate model and then use it to determine whether or not the rise in global mean temperature has had a significant impact on the climate in Leicester. Then, a recommendation as to whether there is an imminent need (within the next ten years) for investment in adapting buildings to cope with a changing climate will be made.

The paper can be divided into three parts. The first part is an expository piece that will construct a model. The model itself is an energy balance model (EBM) in the form of an ODE, for which the equilibrium points will be found. The second part will then implement this model using data collected from the years 2002 to 2019 inclusive. The data is analysed using the Wilcoxon test, with its use justified by the variance test. Finally, a brief overview of the significance of climate modelling and the frontier climate science research is given.

This paper is suitable for undergraduate students with limited exposure to the applications of Mathematics to climate change. As such, the model being used in this paper is a gross over-simplification of the real climate situation and serves only to introduce the concept of climate modelling and its applications in a familiar setting.

1 Introduction

1.1 What is climate?

We often think of climate as being synonymous with weather. However, there is a difference and it is vital that we distinguish between the two. The difference between weather and climate is a measure of time. Weather is what conditions of the atmosphere are over a short period of time, i.e. one day, and climate is how the atmosphere "behaves" over relatively longer periods of time, i.e. three decades¹.

In order to model the climate, we need to understand how it actually works. The *climate system* consists of five main sub-systems:

- atmosphere: air, including all gases and water vapour;
- hydrosphere: water, i.e. oceans, seas and large bodies of water;
- cryosphere: ice, i.e. glaciers, polar ice caps;
- lithosphere: land, i.e. rocks, sand, soil;
- biosphere: living material, i.e. flora, fauna, humans.

¹Definition taken from [6].

All of this is powered by solar radiation and evolves under the influence of individual characteristics, ocean currents, air flow and external factors, called forcings. Forcings can take various forms, such as natural phenomena; variations in solar output, and human induced (anthropogenic) factors: land use, burning of fossil fuels, etc. (see figure 1 below).

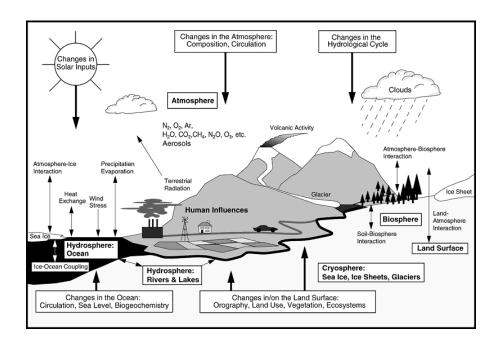


Figure 1: An overview of Earth's climate system. Credit: [2, figure 1.1, pp. 88]

1.2 What is a model?

A mathematical model takes a real life situation and employs assumptions to simplify the situation to just science, from which equations can be derived in an attempt to predict what may happen, given incremental fluctuations in variables. Models are also very useful in optimisation problems. But what matters most in this paper is a model of the climate so we can determine whether or not the fluctuations in climate that we are currently experiencing in Leicester, are statistically significant.

$\mathbf{2}$ The Model

Energy Balance Model (EBM)

The model we will be using is the energy balance model (EBM) [see 3, pp. 16-21]. The EBM follows the basic physical law of conservation of energy. However, it is not an accurate model. The EBM is characterised by the following

$$C\frac{dT}{dt} = E_{in} - E_{out}, (1)$$

where C is the specific heat capacity of the Earth averaged over the entire globe; T is the global mean temperature; t is time; and E_{in} , E_{out} are the amount of energy received and radiated by the Earth, respectively. This is a non-autonomous ODE. To find the equilibrium points³ of this ODE, we set (1)

²See [11, table 2.1] for a more in-depth discussion on the accuracy of models and how the accuracy is measured.

³We want to find the equilibrium points so that we can later find the range of acceptable values for the global mean temperature of the Earth.

to be equal to zero, that is, $C\frac{dT}{dt} = 0$, or equivalently

$$E_{in} = E_{out}$$
 (2)

The oceans (covering 70% of the Earth) store most of the Earth's energy, since water has a very high specific heat capacity. Although the consideration of specific heat capacity of the Earth in climate models is arguably essential [see 4], we are only looking at Leicester—which is mostly land—hence, we will set the specific heat capacity to be equal to zero.

2.2Determining total incoming energy

The next challenge is to find expressions for E_{in} and E_{out} . To do this, we need to make some assumptions. When viewed from the Sun, the Earth can be modelled as a circle⁴, with area πR^2 . The amount of solar radiation that hits the Earth's surface is called the incident solar radiation (insolation), and is denoted by S_0 . Thus, the total insolation hitting the Earth is $\pi R^2 S_0$.

However, it is inevitable that some fraction of insolation will be reflected back into space before it even reaches the Earth. This could be caused by many different things, like space debris and cloud coverage. This fraction α , is called the *albedo*, and we account for this by setting insolation to be $(1-\alpha)S_0$, $(1-\alpha)$ is called the co-albedo). Up until now, we have only been considering the patch of the Earth that is facing the Sun at any given time. This patch is a quarter of the Earth's total surface area. To account for this, we must divide by the total surface area of the Earth, which is $4\pi R^2$. Putting all of this together, we now have the total insolation,

$$E_{in} = \frac{1}{4}(1 - \alpha)S_0. {3}$$

2.3Determining total outgoing energy

The Earth emits electromagnetic radiation in the infra-red region of the spectrum. We will assume that Earth is a black body, that is an idealised physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. Let us say that the global average temperature of the Earth is T in $Kelvin^5$. The average amount of energy radiated per unit area per unit time follows the Stefan-Boltzmann Law, that is $E_{out}: T \longrightarrow E_{out}(T)$ such that

$$E_{out}(T) = \sigma T^4, \tag{4}$$

where $\sigma = 5.67 \times 10^{-8} \ Wm^{-2}K^{-4}$ is Stefan's constant⁶. We now have a model that can be used to process our data

$$C\frac{dT}{dt} = \frac{1}{4}(1 - \alpha)S_0 - \sigma T^4,\tag{5}$$

with equilibrium solution

$$\frac{1}{4}(1-\alpha)S_0 = \sigma T^4. \tag{6}$$

However, this is not a very good model, as will be explained next.

⁴We can ignore the inverse square law of solar radiation here, because the radius of the Earth is negligible when compared with the distance between the Sun and Earth.

⁵Kelvin uses the same scale as degrees Celsius, with the translation K = C + 273.

⁶One unit of Stefan's constant is a Watt per metre squared per Kelvin to the fourth power. However, the details of this are not necessary for constructing the model.

Modifying the model 2.4

Accounting for Greenhouse gas emissions 2.4.1

There are some obvious limitations to this model, since we are not accounting for factors like wind speed, ocean currents, natural phenomena (e.g. volcanic eruptions, forest fires etc.) to name a few. However, it is possible for us to include the greenhouse effect in our model. Particles (of CO_2 , water vapour, methane, aerosols, etc.) in the air block the mainly infra-red radiation emitted by the Earth from escaping into space. We call these greenhouse gases (GHGs). This leads to the heating of the Earth. To maintain an energy balance, we must compensate for this increase in temperature. The easiest way to do this would be to reduce the expression for E_{out} by a factor ε , where $0 < \varepsilon < 1$. So now,

$$E_{out}(T) = \varepsilon \sigma T^4, \tag{7}$$

the EBM is

$$C\frac{dT}{dt} = \frac{1}{4}(1 - \alpha)S_0 - \varepsilon\sigma T^4,\tag{8}$$

with equilibrium solution

$$\frac{1}{4}(1-\alpha)S_0 = \varepsilon\sigma T^4. \tag{9}$$

This is a very subjective solution to this problem, since we can only guess what the value of ε must be, but it is still better than not accounting for GHGs at all.

Accounting for the changing of state of water 2.4.2

Another less obvious limitation of this model is the fact that water can change state to snow and then ice, depending on temperature. This changing of states can be accounted for through modifying the value of the albedo, α . Since the temperature affects the state of water, we can write $\alpha: T \longrightarrow \alpha(T)$, that is albedo as a function of temperature. The typical value of Earth's albedo over water is $\alpha = 0.3$, and $\alpha = 0.7$ over ice, thus

$$\alpha(T) \approx \begin{cases} 0.7 & \text{if } T < 250K \\ 0.3 & \text{if } T > 280K \end{cases}$$
 (10)

which can be generalised to

$$\alpha(T) = \frac{1}{2} - \frac{1}{5} \tanh\left(\frac{T - 265}{10}\right),\tag{11}$$

(where the factor 10 just ensures the values produce a 'nice' graph)⁷. The incoming energy is now a function of the mean surface temperature,

$$E_{in} = \frac{1}{4}(1 - \alpha(T))S_0, \tag{12}$$

and the total outgoing radiation is as in (7).

2.4.3 Budyko's model

This model does not use satellite data to improve its accuracy. A slightly more refined model, first proposed by Mikhail Budyko accounts for this. However, we will not be using this model since the satellite data needed is not readily available to us.

⁷How we obtain this function for the albedo is irrelevant to our discussion. For a slightly more detailed explanation, see [10, pp. 120].

2.5 Final model

Putting together (12) and (7) into (1), leads us to the final EBM that we will use to process the data.

$$C\frac{dT}{dt} = \frac{1}{4}(1 - \alpha(T))S_0 - \varepsilon\sigma T^4,$$
(13)

with equilibrium solution

$$\frac{1}{4}(1 - \alpha(T))S_0 = \varepsilon \sigma T^4 \,. \tag{14}$$

Equilibrium points of this model 2.6

We would like to know for which values of T our model has solutions so that we know what the range of acceptable values for the global mean temperature is. These solutions are called equilibrium points and to find them, we must solve (14). This cannot be done analytically, so a solution needs to be found numerically. We will do this by plotting graphs using MATLAB and looking for any point(s) of intersection.

The first function plots the incoming energy (with units Watts per square metre, Wm^{-2}) with respect to temperature (in Kelvin, K):

```
function [e_in] = energy_in_plot(T)
2
     S0 = 1368;
3
     e_{in} = (1/4)*(1 - (0.5 - 0.2*tanh((T-265)/10)))*S0;
4
          plot(T, e_in, 'r')
5
          title('Energy as a function of time')
6
          xlabel('Temperature (K)')
          ylabel('Energy (Wm^{-2})')
 end
```

The second function plots the outgoing energy (with units Watts per square metre, Wm^{-2}) with respect to temperature:

```
function [e_out] = energy_out_plot(T)
2
      epsilon = 0.6; %accounting for GHG
3
      sigma = 5.67*10.^(-8); %Stefan's constant
4
      e_out = epsilon*sigma*T.^(4);
5
          plot(T, e_out, 'b.')
          title('Energy as a function of time')
6
7
          xlabel('Temperature (K)')
8
          ylabel('Energy (Wm^{-2})')
 end
```

Now, in the command window to obtain the plot:

```
>> T = linspace(200,310);
>> energy_out_plot(T);
>> hold on
>> energy_in_plot(T);
```

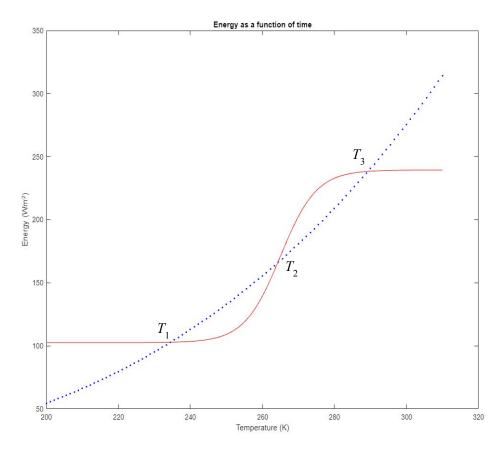


Figure 2: This graph shows the equilibrium points for total incoming and outgoing energy for the Earth

The points of intersection are the equilibrium points, which can be read off relatively easily from figure 2. There are three equilibria at (read from left to right):

$$T_1 = 234.4K = -38.6^{\circ}C,$$
 $T_2 = 264.4K = -8.6^{\circ}C,$ $T_3 = 288.9K = 15.9^{\circ}C.$

This means that the Earth can have multiple equilibria. T_3 corresponds to the stable current world climate. This point is called a stable equilibrium because if we look at figure 2, we will see that as the temperature increases, the outgoing energy (blue, dotted line) goes above the incoming energy (red, solid line). This makes the right hand side of the EBM (13) negative. So the overall temperature will decrease with respect to time, bringing the global mean temperature back down to T_3 . A similar sort of thing would happen if the temperature decreased slightly. The global mean temperature is approximately $14.753^{\circ}C$ today. This is not to say that climate change is not really happening, but rather that our model (14) is not very accurate. Hence, this is a good enough match with T_3 . In fact, small fluctuations in the global mean temperature—even just $1^{\circ}C$ —can alter the climate quite drastically. This is because the insolation received by the Earth rarely varies.

The next two equilibria are not entirely necessary for the analysis process, but a brief explanation of each is given for completeness. At T_2 , we have what is called an *unstable* equilibrium. This is because if the temperature were to increase slightly from T_2 , looking at figure 2, we can see that the incoming energy would be greater than the outgoing energy, so the right hand side of the EBM (13) would be positive and the overall temperature will increase with respect to time and thus move rapidly away from T_2 until it reached the stable equilibrium at T_3 . A similar thing would happen if the temperature were to decrease slightly from T_2 , except the temperature would decrease rapidly until it reached the stable equilibrium at T_1 .

The stable equilibrium at T_1 is very interesting, because it indicates that the Earth can be, or may have been in a state of deep freezing at some point. In other words, the earth could be covered entirely in snow and ice. This type of scenario is commonly referred to as *Snowball Earth*. There exists a growing amount of evidence that Snowball Earth happened around 715 million years ago, and lasted for approximately 120 million years. The precise state of the Earth during this time is still under debate, but many scientists today believe it to have happened.

3 Using the EBM

3.1 The statistical analysis process

In the world of climate modelling, a column of atmosphere is studied at any given time. These columns are called 'noodles'. We will be looking at the noodle of Leicester, and using temperatures recorded in Leicester everyday for the last eighteen years⁸, and considering each month separately. This is because the temperature between months fluctuates due to the changing of seasons.

We will process the data (average temperature in Leicester each month from 2002 to 2019 inclusive) in two parts.

Part 1: Analysing the temperature data to provide a comparison for the results of our model

- (i) Split the monthly temperature averages into two groups: the 'old' years 2002 to 2010 inclusive, and the 'new' years 2011 to 2019 inclusive, for each month,
- (ii) Perform a normal diagnostic test to check if the data follows a normal distribution,
- (iii) Check that the variances of the two groups are equal with a variance test,
- (iv) Perform a Wilcoxon test in RStudio, with:
 - (a) a confidence level of 0.9,
 - (b) null hypothesis: the mean temperature \overline{T} for old years is equal to \overline{T} for new years, that is $H_0: \overline{T_{old}} = \overline{T_{new}}$,
 - (c) alternative hypothesis: the mean temperature \overline{T} for old years is **not** equal to \overline{T} for new years, that is $H_1: \overline{T_{old}} \neq \overline{T_{new}}$,

Part 2: Analysing the results of our model

- (i) Calculate $\Delta = E_{in} E_{out}$ for every month from each year,
- (ii) Split the data into two groups as above,
- (iii) Perform a normal diagnostic test to check if the data follows a normal distribution,
- (iv) Check that the variances of the two groups are equal with a variance test,
- (v) Perform a Wilcoxon test, with:
 - (a) a confidence level of 0.9,
 - (b) null hypothesis: the $\overline{\Delta}$ for old years is equal to the $\overline{\Delta}$ for new years, that is $H_0: \overline{\Delta_{old}} = \overline{\Delta_{new}}$,
 - (c) alternative hypothesis: the $\overline{\Delta}$ for old years is **not** equal to the $\overline{\Delta}$ for new years, that is $H_1: \overline{\Delta_{old}} \neq \overline{\Delta_{new}}$,

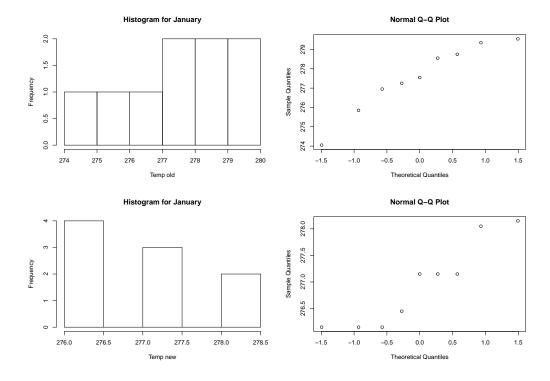
⁸Data obtained from [7], [8] and [9].

3.1.1 Analysis for January

Now, we shall perform this analysis for the average temperature in January over the last eighteen years using RStudio. We begin by running the *normal diagnostic test* to see if the temperatures for each of the groups 'old' and 'new' follow a *normal distribution*. To do this, we draw plots of the data. The hist (lines 2, 4 below) command draws a histogram of the data and the qqnorm (lines 3, 5 below) command draws a normal Q-Q plot:

We are looking for a 'bell-shaped' histogram and an approximate straight line Q-Q plot. From figure 3, we can see that none of the graphs fit these criteria.

Figure 3: This shows the histogram and normal Q-Q plots for average temperatures in January



To account for this non-normality, we perform a Wilcoxon test. The Wilcoxon test performs a t-test with non-parametric (i.e. non-normal) data. We reject the null hypothesis $H_0: \overline{T_{old}} = \overline{T_{new}}$, if the confidence interval does not contain zero. However, we must justify use of the Wilcoxon test. To do this, we perform a variance test to ensure that the data satisfies the requirements for a Wilcoxon test. The variance test identifies whether the variances differ from each other significantly or not. We reject the null hypothesis H_0 : the ratio of variances is one (i.e. the variances are equal) if the confidence interval does not contain 1:

```
> var.test(Temp~Type, data=dataJan, conf.level=0.99)
1
2
3
    F test to compare two variances
4
5
      data:
              Temp by Type
6
      F = 0.1955, num df = 8, denom df = 8, p-value = 0.03296
7
      alternative hypothesis: true ratio of variances is not equal to 1
8
      99 percent confidence interval:
9
        0.02608055 1.46542950
10
      sample estimates:
11
      ratio of variances
12
                0.1954973
```

The confidence interval (line 8) of the variance test contains 1, so we can assume that the variances are equal with 99% confidence. We now use the Wilcoxon test:

```
> wilcox.test(dataJan$Temp~dataJan$Type, conf.int=TRUE, conf.level
          =0.9)
2
3
    Wilcoxon rank sum test with continuity correction
4
5
              dataJan$Temp by dataJan$Type
      data:
6
      W = 27, p-value = 0.249
7
      alternative hypothesis: true location shift is not equal to 0
8
      90 percent confidence interval:
9
        -2.1000519
                    0.3000299
10
      sample estimates:
      difference in location
11
12
                     -0.800023
```

It can be seen from the confidence interval (line 8) of the Wilcoxon test that we have no reason to reject the null hypothesis, $H_0: \overline{T_{old}} = \overline{T_{new}}$. From this analysis we can conclude, with 90% confidence, that the average temperature in January for the years 2011 to 2019 were not any warmer (or cooler) than January in the years 2002 to 2010.

3.2 Analysis in RStudio for all months

A similar process follows, as outlined by **Part 2** above, to analyse the change in $\overline{\Delta}$. The RStudio code remains the same, so is omitted. We can repeat this process for all other months in the same way. The normal diagnostic test for all months showed that none of the data follow a normal distribution. Furthermore, the variance test shows all the data to have equal variances with 99% confidence. Hence, we proceed with the Wilcoxon test. To save space, only the results are shown.

	var.test confidence interval		wilcox.test	
Month			confidence interval	
	lower	upper	lower	upper
January	0.02608055	1.46542950	-2.1000519	0.3000299
February	0.507893	6.003570	-1.300015	1.399996
March	0.7071984	39.7364910	-1.699974	1.000055
April	0.857643	10.137801	-1.999976	0.300010
May	0.840056	9.929920	-1.000030	0.700048
June	0.1626469	9.1389039	-1.700074	-0.300034
July	0.359159	4.245455	-1.300058	1.400045
August	0.411939	4.869337	-1.100053	0.699969
September	0.349937	4.136440	-1.700078	0.399998
October	0.252188	2.980996	-0.900041	1.700051
November	0.680873	8.048282	-1.300059	1.299950
December	0.132426	1.565344	-0.000012	2.900006
All Months	0.389301	4.601753	-0.780000	0.480000

Table 1: Results of the tests for the monthly average temperatures.

Table 2: Results of the tests for $\overline{\Delta}$.

	var.test		.test wilcox.test	
Month	confidence interval		confidence interval	
	lower	upper	lower	upper
January	0.02843767	1.59787320	-0.042771	2.056882
February	0.563121	6.656399	-1.491278	1.270834
March	0.5561304	31.2481939	-1.985310	2.288222
April	0.831690	9.831020	-0.743644	4.236719
May	0.825951	9.763183	-2.007860	2.725502
June	0.335594	3.966908	0.934649	5.091377
July	0.347665	4.109591	-4.489421	4.103029
August	0.397794	4.702141	-2.209623	3.625140
September	0.317782	3.756352	-1.233949	5.157078
October	0.247761	2.928665	-4.549335	2.560250
November	0.903909	10.684697	-2.647989	2.495453
December	0.5090681	28.6038262	-3.1833564	0.3295915
All Months	0.411263	4.861352	-1.762400	2.361100

Conclusions 4

Interpreting the results

From table 1, we can conclude with 90% confidence that there is no statistical difference in the values $\overline{T_{old}}$ and $\overline{T_{new}}$ for all months except June.

We shall take a closer look at June by comparing $\overline{T_{old}}$ with $\overline{T_{new}}$.

Table 3: This table shows the values for $\overline{T_{old}}$ and $\overline{T_{new}}$ for June.

June		
$\overline{T_{old}}$	$\overline{T_{new}}$	
15.06	14.10	

In table 3, it can be seen that the average temperature in June has decreased from the 2002-2010 period to the 2011-2019 period.

From table 2, we can conclude with 90% confidence that there is no statistical difference in the values $\overline{\Delta_{old}}$ and $\overline{\Delta_{new}}$ for all months except June.

We shall take a closer look at June by comparing $\overline{\Delta_{old}}$ with $\overline{\Delta_{new}}$ and determining whether there has been a net increase or decrease in the value $\overline{\Delta}$ across nine years.

Table 4: This table shows the values for $\overline{\Delta_{old}}$ and $\overline{\Delta_{new}}$ for June.

June		
$\overline{\Delta_{old}}$	$\overline{\Delta_{new}}$	
-4.4658	-1.5438	

It can be seen from table 4 that $\overline{\Delta}$ has increased from the 2002-2010 period to the 2011-2019 period. Since $\Delta = E_{in} - E_{out}$, and the values $\overline{\Delta_{old}}$ and $\overline{\Delta_{new}}$ are both negative, we know that $E_{out} > E_{in}$. It is uncertain whether E_{in} has increased or E_{out} has decreased to produce this significant change in value. However, it is clear that there has been an increase in the energy retained in the month of June. This is inconsistent with our analysis of the mean temperature data, which showed a net decrease in temperature.

What does this mean for Leicester? 4.2

The purpose of this paper is to provide a recommendation to the Leicester City Council with regards to adapting buildings to cope with a changing world climate. The results of the analysis provide little evidence for climate change having a significant impact on the climate in Leicester. Therefore, my recommendation is that there is no imminent need for any significant adaptations of buildings.

The aforementioned inconsistency between analyses could suggest that the increased energy was used in the evaporation of water, rather than raising the average temperature, which could contribute to increased cloud coverage. This would explain the decrease in average temperature in June, but an increase in overall energy retention in June. However, the actual cause of this discrepancy may be attributed to other factors not considered in this model, such as cloud coverage; wind speed and direction; and specific GHG emissions. Specific studies would be required to confirm or deny this possibility.

What are the limitations of this model? 4.3

The EBM models climate change by taking the global mean temperature as input. The global mean temperature varies very little over time⁹. We are using the average monthly temperatures of Leicester,

⁹There is substantial evidence [see 5] to suggest that the global mean temperature is currently increasing rapidly when compared with the variation in the last century.

which is a very small area of the Earth and temperature fluctuates a lot more each month than the global mean temperature. Because of this, our model is not efficient for what we set out to accomplish.

Furthermore, our data spans only the last eighteen years. Since the EBM measures changes in energy over hundreds of years, we will not get an accurate portrayal of climate change in Leicester. A possible way around this may be to collect average temperatures spanning say, a hundred years. This would likely be a more reliable analysis. There are two problems with this: firstly, with so much data, we would need a lot more time and computer processing power to be able to analyse it all; and secondly, the data of average monthly temperatures in Leicester is not readily available.

Clouds are a major cause for the inaccuracies in all climate models. Even today, climate modellers with the latest technology cannot produce more definitive climate predictions. Most models use data collected from 'grid cells', that is the Earth's surface area is divided into boxes of about $12,000\,\mathrm{km}^2$ each. These boxes are then projected $100\,\mathrm{km}$ into the sky and about $20\,\mathrm{km}$ into the ground, called 'noodles'. The average cloud spans about $16\,\mathrm{km}^2$ of the Earth's surface. Clouds can only form if there is enough heat to produce the water vapour necessary, but clouds reflect sunlight and significantly reduce the amount of energy that reaches the Earth's surface to warm it (despite their relatively small area). This seems contradictory, which is why clouds are an active area of climate research.

5 Why is climate science important?

There is a lot of controversy surrounding climate change and this is due to many factors. The most vocal climate change deniers come from those with huge economic power. Whilst some economies may benefit from implementing sustainable practice, a large proportion of economies would collapse. The coal, natural gas and crude oil industries would all cease to exist if renewable energy became the standard across the globe and stakeholders in such industries would no longer benefit.

However, there are sustainable ways around this issue. Cars have not changed much since their invention in 1885, until recent research into electric vehicles prompted their mass production. Oil is needed to make fuel for cars, but with the advent of renewable energy, electric cars can be charged and run with much less carbon emissions compared with traditional cars.

There are numerous studies, like this one, providing real scientific evidence in favour of immediate action to reduce carbon emissions. Although not a lot of the models used could be far more accurate, the evidence still overwhelmingly suggests that inaction will cause irreversible damage to the Earth's natural balance. Such a disruption would be detrimental to many things, including human health. It is in the best interests of every human being to individually reduce their personal carbon emissions to ensure a safe and certain future for the next few generations.

The people at the frontier of climate research are playing their part by seeking to understand cloud formation and turbulence in relation to air flow and ocean currents. This will enable climate modellers to make more accurate predictions of the Earth's climate, which then should not be ignored.

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