Game Theory Behind Shop Competition

Lewis Pledger

8th March 2019

Abstract

This paper covers game theory at a basic level and looks at an elementary application of game theory, the prisoner's dilemma, in two different ways. The paper goes on to use the example of competitive pizza shops and how game theory can help to make the best business decisions when it comes to whether to advertise or not and how to price slices of pizza. This is an interesting topic that is quite new in the mathematical world. Since John von Neumann and Oskar Morgenstern invented the idea in 1944, there has been a lot of progression in this field with lots of applications. With John Nash being one of the most well-known game theorists to date. John Nash found 'the Nash equilibrium' which is mentioned within this paper.

Introduction

Individuals make decisions on a daily basis. Game theory, also called the strategy theory or the theory of interactive decision making, analyses how individuals and companies make decisions by comparing what they get or the outcome. Even though it has applications in a lot of fields, it is mainly applied in economics to determine the outcome of economic transactions by applying the utility theory to model the interests of the players. The theory states that players are most likely to use a strategy that will maximise their utility (Leyton-Brown and Shoham, 2008, p.1). Among the most renowned applications of game theory is the prisoner's dilemma. It is mainly used in determining the choices that different parties make when subjected to strategic situations or conditions with similar rules and outcomes by applying mathematical models. The term strategic situations refer to a situation in which two parties try to make decisions that will favour them by trying to anticipate the reaction of the other party in a similar situation (Rauhut, 2015). It is applied in a lot of fields, but more so in economics to analyse any strategic interaction between two parties. This paper will introduce the readers to the game theory by enlightening them on some common terms that will enhance their understanding of game theory, assumptions, the prisoner's dilemma in more details, looking at an example including two different pizza shops and conclude with some of the maths behind finding the 'Nash equilibrium'.

Terms

It is important to familiarise with the terms that are frequently used in game theory.

- "The game" refers to a set of circumstances whose outcome is dependent on the actions of two or more parties that are making the decisions.
- "Players" are the parties that are involved in a game, and they make decisions that are within the rules and outcomes of the game.
- "Strategy" refers to the plan of action that any given player in the game will take given the rules and circumstances provided in the game (McCain, 2014, p.5).
- "The payoff" refers to the payout, or what any player receives after making certain choices within the context of the game. The payoff may be in a monetary form or in the form of a utility if it is in a quantifiable form.
- The "information set" refers to the data available to each player at a given point in the game.
- "Equilibrium" is another term that is used to refer to the final outcome that is arrived at after all the parties involved make a decision.
- The "Nash equilibrium" is a concept in which the players have no motive to deviate from their chosen strategy even after considering the choice taken by the opponent since they cannot gain any additional benefit (Tandelis, 2013, p 81).

Just like every other theory, game theory also has its assumptions. The first assumption is that the parties involved in the game practice instrumental rationality that assumes that each player tries to maximise their utility under the given rules (Bier & Azaiez, 2009, p.21). Additionally, it assumes the following: that the player can adopt multiple strategies to solve a problem, that the players are well informed about the rules and the outcomes of other players and that there are pre-determined outcomes for the players. The theory also assumes that there is a conflict of interest among the players.

Elementary Applications

Game theory has a lot of applications in modern life. There are numerous games. However, the applications are mainly based on the three fundamental categories of games. The first fundamental category is the sequential games in which the players decide after each other and they at least have some knowledge on the previous move made by the other party. Trust games, which are a subset of sequential games are widely used when it comes to where to place and honour trust. Common examples of their application in the modern world are when it comes to delaying payments and sending defective goods and taxi drivers who risk being deceived or assaulted by bad customers (Rauhut, 2015 n.p).

The second fundamental category is games with either perfect or imperfect and complete or incomplete information. In games with complete or perfect information, the players are provided with all the information they need to decide their next move. Games with incomplete information are characterised by a move in nature in which the types of players are determined, or the game structure is determined (Myerson, 2013, p.44).

There are also cooperative or non-cooperative games. Cooperative games are characterised by pre-determined joint action agreements that are enforceable through a third party. Non-cooperative games, on the other hand, allow individuals to act on their self-interest as there is no enforcement of the laws of the game (Nitisha, 2019 n.p). The last category is the zero-sum and positive sum games. In a zero-sum game, one player's gains or losses are equally balanced by what the other player loses or gains. Positive-sum games, also called "win-win" games are constructed in such a way that every player can benefit from the transactions.

The Prisoner's Dilemma

The most common example of an application of game theory is in the prisoner's dilemma.

With Respect to Confessions of Feelings

		Partner one	
		Confess feelings	Don't confess feelings
Partner two	Confess feelings	Romantic escapades	One feels rejected
	Don't confess feelings	One feels rejected	No one gets rejected

Fig. 1 (Jameson, 2014 p.16).

In the modern world, the prisoner's dilemma can be used by individuals who have feelings for each other, but they fail to muster up the courage needed to confess their feelings to each other. In the above example, partner one chooses the column while partner two chooses the row. The words in each cell are used to represent the payoff for each choice of strategy by either of the two partners. In the first column, if both confess their feelings, then the two partners will go ahead to develop a relationship that will bring each of them a lot of joy.

In the second column, one partner confesses his/her feelings while the other one is too shy to confess their feelings, or they even don't have the feelings. The payoff for this choice by the first partner will lead to a feeling of rejection that is not desired in any way. The same happens if the two partners switch roles and one of the partners decides to express themselves while the other partner chooses to remain silent. The one partner who decided to confess their feelings will end up feeling rejected, an outcome that is rarely desired by anyone.

In the last scenario, both partners decide to remain silent about their feelings and therefore no one ends up being rejected. Such is the dominant strategy since none of the partners will feel rejected regardless of the choice that the other partner would have taken. Even though the option of not confessing is the dominant strategy according to game theory, it rarely brings the most desired outcome since one only gets to protect their feelings from being hurt just for a while but then they get to settle for less by allowing the person they really had feelings for to go.

In that scenario, the best option is to confess one's feelings instead of settling for less. Although this is a nice use of game theory, there are lots of areas for emotion which slightly detracts from the maths. Below I will give another example using the idea of the prisoner's dilemma with actual prisoners.

With Respect to Court Confessions

		Riggz	
		Confess	Don't confess
Bob	Confess	5 years, 5 years	0, 10 years
	Don't confess	10 years, 0	2 years, 2 years

(Jameson, 2014 p.16).

In the prisoner's dilemma, Riggz chooses the column while Bob chooses the row. The numbers in each cell are used to represent the payoff for each choice of strategy by either of the two prisoners. The number to the right of the comma in each cell shows the payoff for the player who chooses the column (Riggz) while the one to the left shows the payoff for the one who chooses the row (Bob). In the first column, if both confess, then they get five years in jail. If Riggz confess and Bob fails to confess, Bob walks free while Riggz is jailed for 10 years. In the second column, if Riggz don't confess and Bob confesses, then Bob is jailed for 10 years while Riggz walks free. If they both fail to confess, then they are jailed for 2 years. Therefore, it is the best strategy is both to confess since none of the players is aware of the strategy that the other player is going to take.

Competitive Pizza Shops

The following is an example of how game theory can be used in a business scenario. In the example below, the managers of the two firms, firm A and firm B, are using the theory to determine whether to advertise based on the payoffs.

		Firm A	
		Advertise	Don't advertise
Firm B	Advertise	4,4	10,2
	Don't advertise	2,10	8,8

Fig. 2 (Prisner, 2014, p.5)

In the above example, if firm A decides to advertise and B does not, then it gets a payoff of 10. If A decides to advertise and B also advertise, then it gets a payoff of 4. If however, it chooses not to advertise and firm B advertises, then it gets a payoff of 2. If however firm B also chooses not to advertise, then the payoff is 8. When it comes to the case of firm B, if it chooses to advertise and A does not advertise, then the payoff is 10, but if A also chooses to advertise, then it gets a payoff of 4. If however, it chooses not to advertise and firm A advertises, then it gets a payoff of 2, while if both firms chose not to advertise, it gets a payoff of 8. Since neither of the firms is sure whether the rival will advertise, they will both chose to advertise as they will get a payoff of 4 but if they fail to advertise, they will have a smaller payoff of 2. Therefore, the best alternative is to advertise.

Since pizza can be said to be a favourite meal for many people, applying the theory to determine how various restaurants come up with the prices of pizza can serve as a good example in addition to offering insight to pizza lovers on how the prices of pizza are determined. Let us take, for instance, there are two restaurants that charge different prices for a slice of pizza, either £1, £2, or £3. In a given month, it is expected that 10,000 pieces of pizza are taken by tourists while the locals take 8,000 pieces and they prefer to go to the restaurant with lower prices. In case the prices are equal, then they split evenly between the two restaurants.

If restaurant A charges £1 for and B charge £2, then all the locals will buy their pizza from restaurant A. Therefore, it will serve 5000 pieces to tourists and 8000 pieces to the locals that will result in 13,000 pieces of pizza sold. As a result, the restaurant will earn £13,000 while B will only serve 5000 pieces to tourists and therefore the restaurant will earn £10,000. The payoff matrix for the two restaurants will be as shown below in 'Fig. 3'. The payoff matrix gives the money earned with it reading (B,A), meaning that the value to the left of the comma is the payoff for restaurant B and to the right of the comma is the payoff for restaurant A.

(figures in **bold** are in thousands)	Restaurant A			
	(B,A)	£1	£2	£3
Restaurant B	£1	**9,9**	**10,18**	**15,23**
	£2	**10,13**	**18,18**	**15,26**
	£3	**15,13**	**18,15**	**27,27**

Fig. 3

For each restaurant, making the price of a slice of pizza £2 strictly dominates over £1 since it has a higher payoff for both the restaurants. Therefore, the price £1 can be eliminated for both restaurants. Upon eliminating the price of £1, the payoff matrix will be as shown below.

(figures in **bold** are in thousands)	Restaurant A		
	(B,A)	£2	£3
Restaurant B	£2	**18,18**	**15,26**
	£3	**18,15**	**27,27**

Fig. 4

Eliminating the move will lead to a reduced game. Upon removing move £1 and remaining with moves £2 and £3, if restaurant B chooses £2, A will never choose £3 since it has a lower payoff of 15, 27 as compared to 2 that will be 18, 26, therefore £2 will dominate £3 for restaurant A. Therefore, move £3 can be eliminated for each restaurant to remain with move £2 that will lead to a payoff of £18,000 for each restaurant and therefore they will choose the price of the pizza to be £2. Move £2 can be said to be dominant. The above example if from dominated moves that tells the player what not to play instead of what to play.

Cournot Competition/Nash Equilibrium

The game theory is also applied in Cournot competition, in which two competing companies producing the same goods within an industry determine the number of goods to produce independently and simultaneously (Talwalker, 2014 n.p). If they produce too much, the market will be flooded and therefore the prices of their goods will fall that will lead to a reduction in their profits. Taking the price equation as $p = 1 - q_1 - q_2$ and the firm's objective is to maximise profits. For both firms, the profits will be equal to the quantity they supply in the market. The market price is determined by the quantity supplied by both firms ($q_1$and q_2). Therefore, the profit function of firm A will be:

$$Firm\ A\ (Profit) = (quantity\ produced)\ (market\ price)$$

$$profit(\pi)1(q_1 - q_2\ ) = (quantity\ produced\ by\ A)\ (market\ price)$$

$$\pi 1(q, q_2\ ) = q_1(1 - q_1 - q_2\ )$$

to maximise the profits, the firm will have to satisfy the first order condition which is the derivative of the profit function with respect to q. profit is maximised when the first order condition is equal to 0, as illustrated below.

$$1 - 2q_1 - q_2 = 0$$

Making q_1 be the subject of the formula:

$$q_1 = (1 - q_2)/2$$

That means for every quantity produced by firm B, then firm A can only maximise profits by producing $[(1 - q_2)/2]$ quantity of the good.

For firm B to maximise profits, its profit function will be

$$Firm\ B\ (Profit) = (quantity\ produced)\ (market\ price)$$

$$profit(\pi)2(q_1, q_2) = (quantity\ produced\ by\ B)\ (market\ price)$$

To maximise the profits, the firm must satisfy the first order condition which will be:

$$\pi 1(q_1, q_2) = q_1(1 - q_1 - q_2)$$

Making q_2 the subject of the formula:

$$q_2 = (1 - q_1)/2$$

Substituting it into firm A's profit equation,

$$q_1 = (1 - q_2)/2$$

$$q_1 = (1 - [(1 - q_1)/2])/2$$

$$2q_1 = 1 - (1 - q_2)/2$$

$$2q_1 = 1/2 + q_2/2$$

$$4q_1 = 1 + q_2$$

$$3q_1 = 1$$

$$q_1 = 1/3$$

$$Therefore, q_2 = (1 - q_1)/2 = 1/3\ as\ well.$$

Consequently, both firms should produce 1/3 of the quantity that the other firm produces to maximise their profits. Since 1/3 is the most optimal strategy and both firms are comfortable with it and they are not willing to change regardless of what the other player chooses, then it can be said to be Nash equilibrium.

There are also very complex forms of game theory, called combinatorial games, for instance in chess. Such games are made up of so many moves that it is hard to analyse them manually. As a result, they can only be analysed using computers due to their complex nature.

Conclusion

Game theory has lots of applications in the day to day lives of individuals. In the social setting, game theory can be used to determine the best strategy to choose, for instance when it comes to deciding whether to dump one's waste locally or to hire a truck to take the waste away. Through the application of game theory in such scenarios, the dominant strategy is always the optimal solution since it always yields a higher payoff regardless of the strategy that the rival adopts. In business situations, the game theory can also be used to guide business decision-making process to ensure that the organisation remains competitive in the market place, like in the competitive pizza shops scenario. Individuals can also apply game theory in making crucial decisions, for instance, while playing rock paper scissors. Additionally, using the game theory, players can arrive at win-win situations, in which both the players gain. It is therefore important to learn and understand game theory and learn to apply it in the day to day activities to gain the most from each interaction that individuals engage in.

References

Bier, V.M. & Azaiez, M.N., 2009. *Game theoretic risk analysis of security threats*, New York: Springer. pp. 21.

Leyton-Brown, K. and Shoham, Y. (2008). *Essentials of game theory*. San Rafael, Calif (1537 Fourth Street, San Rafael, CA 94901 USA): Morgan & Claypool Publishers. Pp1

McCain, R.A., 2014. *Game theory: a nontechnical introduction to the analysis of strategy*, Tuck Link: World Scientific pp. 5-54.

Myerson, R. (2013). *Game Theory*. 1st ed. Cambridge: Harvard University Press, p.44.

Nitisha (2019). *5 Types of Games in Game Theory (With Diagram)*. [online] Economics Discussion. Available at: http://www.economicsdiscussion.net/game-theory/5-types-of-games-in-game-theory-with-diagram/3827 [Accessed 9 Mar. 2019].

Rauhut, H., 2015. Game theory. The *University of Zurich*. Available at: https://www.suz.uzh.ch/dam/jcr:ce4217d7-c808-40a1-ac1a-82cc3c5bf6b7/Rauhut_GameTheory_OxfordHandbook.pdf [Accessed March 8, 2019].

Talwalker, P. (2014). *Game Theory Tuesdays: How Nash Equilibrium, Learning, Differential Equations, and Eigenvalues Are Related – Mind Your Decisions*. [online] Mindyourdecisions.com. Available at: https://mindyourdecisions.com/blog/2014/09/09/game-theory-tuesdays-how-nash-equilibrium-learning-differential-equations-and-eigenvalues-are-related/ [Accessed 10 Mar. 2019].

Tandelis, S. (2013). *Game Theory: An Introduction*. 1st ed. New Jersey: Princeton University Press, pp.50-56,80-83.