A mathematical approach to ranking short deck poker hands and how these rankings could affect play

By Charlie Corden

Abstract

The latest introduction to the family of card games poker has risen to popularity, and so too have discussions of how it should be played. In this paper mathematics and reason are used to address the issue of ranking a Three of a Kind or a Straight higher in the game of short deck (six plus) poker.

1 Introduction

There is a glossary at the end of the paper to explain any poker vocabulary used.
Short Deck poker is similar to Texas Hold 'em [1] or "normal" poker however, the cards two, three, four and five are taken out of the deck of cards. This means that the deck consists of 36 cards, six to Ace of all four suits, this is why it is sometimes called six-plus poker [2]. It is a relatively new game, so there are differing opinions on how to rank the hands compared to Texas Hold 'em. This is clear due to the fact that different versions of the game are played all around the world and not one set of rankings is agreed upon. In this paper, all calculations will be done ignoring the potential cards of the other players at the table as these are unknowns. This is common practice in the poker world for reviewing hands that have been played or assessing potential situations. Including other players' unknown cards would only further complicate calculations, and bring a high degree of error into assumptions, as we could only guess as to what they possess. The primary reason Short Deck poker was invented was for it to be a more fun and faster paced alternative to Texas Hold 'em in which there would be more people involved in each hand and therefore less people not taking part [3]. What this means is that with less cards in the deck it is assumed that with any two cards in your hand you are still likely to end up with a high-ranking hand, essentially eliminating many of the boring hands or hands that were often *folded*. Usually, hands are ranked based on the probability of them occurring, for example, a royal flush {Ten, Jack, Queen, King, Ace} of one suit in Texas Hold 'em is the highest ranked hand because it has the lowest probability of appearing.

Some of these probabilities change when you play Short Deck poker, for example, in Texas Hold 'em if you have {Ten of hearts, Queen of hearts} in your hand, and the *flop* is {eight of hearts, six of hearts, seven of diamonds} then one more heart is needed to make a flush. The probability of this occurring is

$$\frac{9}{47} = 0.1915 \tag{1}$$

as there are 9 remaining hearts in the deck and 47 cards total left in the deck. However, the same situation in a Short Deck game has probability

$$\frac{5}{31} = 0.1613 \tag{2}$$

Therefore, a flush is ranked higher than a full house in Short Deck, this adjustment is made based on probability alone. However, when it comes to assessing which of Three of a Kind or a Straight should be ranked higher, probability alone may not be the most important factor for the good of the game. Consequently, when deciding whether a Three of a Kind or a Straight is worth more it is important to consider the effect on how the game is played and if the game still fulfils its original purpose of encouraging more betting and more exciting hands.

2 Mathematics

In both varieties of poker, you start with two cards in your hand and then end with five other cards on the *board* which can all be used to make your final "hand" at the end of the round. Note for these probabilities we do not count a Three of a Kind made solely of cards on the *board* as a valid outcome. Firstly, the probability of making Three of a Kind in Texas Hold 'em when we start with a pair already in our hands is

$$1 - \left(\frac{48\times47\times46\times45\times44}{50\times49\times48\times47\times46}\right) = 0.1918 \tag{3}$$

This is the probability of none of the two remaining cards in the deck that would make us a Three of a Kind taken away from one, so we're left with the probability that we make a Three of a Kind. Before any cards are dealt, there are only two cards in the deck that can come to make us a Three of a Kind, so for the first card out of the deck the chance that it is not one of those two cards is 48 divided by 50. For each card dealt after this the total number of cards in the deck and the number of cards that will not give us a Three of a Kind go down by one. Hence the numbers on the numerator and denominator decrease sequentially. This same calculation for a Short Deck hand works as follows

$$1 - \left(\frac{32\times31\times30\times29\times28}{34\times33\times32\times31\times30}\right) = 0.2763 \tag{4}$$

After taking out the un-used 30.77% (16) of the cards in the deck, for Short Deck, the chances of a Three of a Kind in this situation has increased from 0.1918 to 0.2763 or by a factor of

$$\frac{2763}{1918} = 1.44056 \tag{5}$$

Also, we would expect the odds of making a Three of a Kind when starting with two different cards in your hand to be lower in both games than the odds above. In Texas Hold 'em, we would calculate this as follows

$$1 - \left(\left(\frac{44\times43\times42\times41\times40}{50\times49\times48\times47\times46}\right) + 2\left(5\left(\frac{3\times44\times43\times42\times41}{50\times49\times48\times47\times46}\right)\right) + 20\left(\frac{3\times3\times44\times43\times42}{50\times49\times48\times47\times46}\right)\right) = 0.0467 \tag{6}$$

In this formula, we have input one minus the probability of not making a Three of a Kind, as this will leave the probability of making the Three of a Kind. Let's take our two cards to be a 2 and a 7 the probability of not making a Three of a Kind is three separate probabilities added together, these are the odds of: no 2 or 7, one 2 no 7, one 7 no 2 and, one 2 and one 7. Once again, these fractions are made by the numerator showing the number of cards in the deck that would give us the outcome we need, i.e. there are 44 cards that are neither 2 or 7, and the denominator showing the total number of cards left in the deck. So, in each fraction the denominator always decreases by one each time a card is drawn from the deck. As having one 2 and no 7 is the same chance as one 7 and no 2 the middle term is multiplied by 2 and there are 5 positions in which the 2 or 7 can occur so we multiply again by 5. There are 20 ways that one 2 and one 7 can appear on the *board* so the final term is multiplied by 20. This leaves us with 0.0467 as the odds for making a Three of a Kind in Texas Hold 'em, starting with two different cards in our hand. A similar calculation from Short Deck gives us 0.0993

$$1-\left(\left(\frac{28\times27\times26\times25\times24}{34\times33\times32\times31\times30}\right)+2\left(5\left(\frac{3\times28\times27\times26\times25}{34\times33\times32\times31\times30}\right)\right)+20\left(\frac{3\times3\times28\times27\times26}{34\times33\times32\times31\times30}\right)\right)=0.0993 \quad (7)$$

So, the chances have increased from 0.0467 to 0.0993 or by a factor of

$$\frac{993}{467}=2.126 \quad (8)$$

This shows that the change of game from Texas Hold 'em to Short Deck has a larger impact on the chances of getting a Three of a Kind with two differing starting cards than it does with a pair.

Something to consider about a Three of a Kind is that if you start with a pair in your hand then you will either make a Three of a Kind or you will not, and if you do not make a Three of a Kind then you still have a pair. This is true for all hands as you cannot have half a Straight, at least it will not count as anything. However, if a player has four cards to a Straight then they are more encouraged to continue in that hand and not *fold*, however, if they do not make their hand then they do not have a "fallback" hand like the Three of a Kind does with a pair. So, in the following calculations, we are considering some probabilities that will show this important role of Straights and how they are often achieved.
We will use an example starting hand of {Ten, Jack} for the following probabilities. The chance that the *flop* will turn your hand into a Straight in Texas Hold 'em is

$$\frac{256}{19600}=0.0131 \quad (9)$$

For this Straight, the *flop* can either be {7, 8, 9}, {8, 9, Q}, {9, Q, K} or {Q, K, A}, for each of these there are

$$4\times4\times4=64 \quad (10)$$

combinations of the three cards as there are four different suits that each of the cards can be. If we add all four of these up

$$64+64+64+64=256 \quad (11)$$

This is the total number of ways that we can make a Straight on just the *flop*. We need to divide this by

$$50Choose3=\frac{50!}{47!\times3!}=19600 \quad (12)$$

which is the total number of potential *flops* that can come out of the 50 remaining cards in the deck and so we get 0.0131. Using the same method for Short Deck and

$$34Choose3=\frac{34!}{31!\times3!}=5984 \quad (13)$$

we get

$$\frac{256}{5984}=0.0428 \quad (14)$$

This outcome is clearly very rare, so we need to look at a more common way of making a Straight, so not all at once on the flop. Let's look at making what we call an open-ended Straight *draw* on the *flop*. This is when, after the *flop* is dealt, we are left with four cards to a Straight and we only need one of the two outside cards to complete the hand, hence open-ended. For example, with our {Ten, Jack} a *flop* of {3, 8, 9} would give us an open-ended Straight draw as we have {8, 9, Ten, Jack} and need a {7} or a {Q} to complete our Straight. The chance that we get an open-ended Straight draw on the *flop* in Texas Hold 'em is

$$\frac{1920}{19600} = 0.09796 \tag{15}$$

The *flop* must contain either {8, 9}, {9, Q} or {Q, K}, and there are 16 combinations of each of those pairings of cards making a total of

$$16 + 16 + 16 = 48 \tag{16}$$

Also, the third card in the *flop* must not complete the Straight otherwise it is not a *drawing* hand, there are 40 remaining cards in the deck that do not complete the Straight. So, our probability is

$$48 \times 40 = 1920 \tag{17}$$

divided by

$$50Choose3 = \frac{50!}{47! \times 3!} = 19600 \tag{18}$$

again or 0.09796. For our Short Deck counterpart, we still need either {8, 9}, {9, Q} or {Q, K}, so 48 combinations but now we only have 24 cards left in the deck that do not give us a Straight, so we calculate

$$\frac{48 \times 24}{34\ choose\ 3} = \frac{1152}{5984} = 0.1925 \tag{19}$$

These probabilities imply that in Short Deck it is more likely to make a Straight than a Three of a Kind however, despite all other hand rankings being solely based on probability many players would argue that a Straight should still be valued higher than a Three of a Kind because of its positive effects on gameplay.

3 Conclusion

Consideration can now be given, using some of our calculations, to assess the impact on gameplay of ranking Short Deck poker hands differently. If a Straight is worth less than a Three of a Kind then a player who is *drawing* towards a Straight is less inclined to continue with their hand as the risk may not be worth the reward, as this is a common occurrence in Short Deck (the chance the *flop* gives an open-ended Straight draw when holding {Ten, Jack} is 0.1925). A game with these rules could dissuade players from entering hands and therefore, lower a player's enjoyment of the game. A similar argument can be made for the reverse, no one will want to play Three of a Kind hands if they are ranked lower however, this does not involve a choice of whether to *draw* to the hand and therefore, is less impactful on gameplay. Any rule changes that encourage more players to partake in each hand increases the likelihood (as a result of more trials, not a higher probability) of rarer hands occurring which would be considered a bonus to the gameplay, be more exciting for the players and consequently achieve the main purpose of the game. Considering that the chance to *flop* a Straight increases by a factor of

$$\frac{428}{131} = 3.27 \tag{20}$$

between Texas Hold 'em and Short Deck implies that players should be encouraged to reach the *flop* with hands that they may not usually want to play.

Looking at styles of play in these games we can see how different rules would change how players play. In these examples we will assume that one player has made a Three of a Kind, one player is *drawing* towards a Straight and the player with a Three of a Kind knows that the other player is *drawing* to a Straight.

Scenario 1 - When a Three of a Kind is ranked higher than a Straight then the player with the Three of a Kind is encouraged to keep the other player in the hand to squeeze as many *chips* off them as they can by not betting or betting small. The player drawing to the Straight will also likely remain in the hand if they end up getting their desired Straight. Both of which result in less aggressive play and overall more players remaining in the hand at the table.

Scenario 2 - When a Straight is ranked higher than a Three of a Kind the player with a Three of a Kind is incentivised to bluff the other player off their hand, resulting in more *chips* changing hands at the table and overall less players in the hand because they might *fold* to these bigger bets. The high percentage chances of Straights or hand *drawing* to Straights occurring in Short Deck show that situations like these would be common and would shape the way the game was played.

When playing Short Deck at a self-organised game, I would suggest that ranking a Three of a Kind higher than a Straight would be more suited to a full ring game of 6 or 9 players as the examples above show that more players would be included in hands. Furthermore, ranking a Straight higher than a Three of a Kind would be better suited to games with only two players as the individual enjoyment of more *chips* being bet in each hand would outweigh the less players being involved in each hand. This is because when a player *folds* in this type of game, the cards are shuffled, and the next hand is played. There is no waiting around and so a more enjoyable game with a bigger emphasis on gambling would be achieved this way.

It is important to note that in any variant of poker it is not just probabilities or hand rankings that would influence decisions such as how to play and when to bet. Many factors go into every decision, such as stack size, player evaluation and position at the table just to name a few. Simply considering what might happen in the game if we change some rules may not actually happen in practice because of all the other factors. Furthermore, it is apparent that the more widely played a type of poker is, the more common the associated styles of play become. Consequently, new styles of play emerge to take advantage of this, which will ensure play is ever changing over the years. Ultimately consideration must be given to the probabilities of certain events when deciding how rankings affect play however, there are many other influential factors at work that should be assessed as individual elements within the game alongside the mathematical approach.

5 Glossary

- Board - All the community cards in a Hold ‘em or Short Deck game - the *flop*, *turn*, and *river* cards together.
- Chips – Counters used to represent money.
- Drawing - To play a hand that is not yet good but could become so if the right cards come.
- Flop - The first three community cards, put out face-up, altogether.
- Fold - To forfeit any chance of winning the current pot in poker, by turning in your cards and sitting out of the current game.
- River - The fifth and final community card, put out face-up, by itself. Also known as "fifth street”.
- Turn - The fourth community card. Put out face-up, by itself. Also known as "fourth street”.

4 References

[1] How to play Texas Holdem Poker - Texas Holdem Rules [Internet]. Pokerstars. 2019 [cited 14 February 2019]. Available from: https://www.pokerstars.uk/poker/games/texas-holdem/

[2] Six-Plus Hold'em Poker Rules - Learn How to Play Six Plus Hold'em | PokerNews [Internet]. Pokernews. 2019 [cited 14 February 2019]. Available from: https://www.pokernews.com/poker-rules/six-plus-hold-em.htm

[3] Rettmuller C. "Six Plus Hold'em" Promoted by Tom Dwan and Phil Ivey [Internet]. PokerTube. 2016 [cited 14 February 2019]. Available from: https://www.pokertube.com/article/114-six-plus-hold-em-promoted-by-tom-dwan-and-phil-ivey?category=poker-gossip-opinion