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The Angular Velocity Required to Rotate a Quintuple Axel

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Abstract

In September 2022, Ilia Malinin landed the first quadruple axel jump in figure skating, performing what was once considered impossible. With all the quadruple jumps having now been landed, the next step for the International Skating Union (ISU) would be to give quintuple jumps base values. After conquering the quadruple axel, a new question arises: is the quintuple axel possible? This paper will explore the physical features of the quintuple axel by using the heights determined from the triple axel in order to predict the angular velocity required.

Keywords: *Sports; Physics; Angular Momentum; Figure Skating; Quintuple Axel*

Introduction

The axel jump, named after its creator Axel Paulson, is the only jump in figure skating to take off forwards, and thus has an extra half rotation compared to other jumps; a double axel has two and a half rotations whereas other double jumps have two rotations [2]. It is an edge jump and does not rely on assistance from the toepick to take off [2] which can help to add height. Since edge jumps do not rely on the toepick but rather pushing against the ice, they are more susceptible to poor ice conditions. This is likely why the quadruple loop jump is not as common as the quadruple flip and lutz jumps. Furthermore, as the axel jump is the only one to take off forwards [2] it can often be the most difficult jump for skaters. Olympic medallist Patrick Chan considered it his weakest jump, saying "[He's] been blessed with good skating skills but not good triple Axel skills" [3].

Assumptions

In order to model the quintuple axel jump, assumptions are required. Data on the quadruple axel is limited, as it is very rarely landed in international competitions [1]. Instead, data on triple axels from the 2019 World Figure Skating Championships [4, 5] has been used as the source. From this data, both the short program and free skate, the average height, distance, and speed of the triple axel will be used (Table 1). The quintuple axel is also five and a half rotations, 1980°, or 11π radians.

Jump	Rotation (◦)	Distance (m)	Height (m)	Speed (km/h)
2A	900	2.24 ± 0.39	0.40 ± 0.05	14.0 ± 4.01
3A	1260	2.83 ± 0.38	0.59 ± 0.05	14.4 ± 3.24

Table 1 – A table showing the mean distance, height, and landing speed of double and triple axels, with the standard deviation as the error. All 3A data is based on the 2019 World Figure Skating Championships men's event [4, 5] and all 2A data on the ladies event [6, 7].

Modelling the Quintuple Axel

Acceleration is the change in velocity over the change in time as described by Eq. 1, the ascent of the skaters jump. At the apex of the jump, the skater is stationary, meaning the final velocity $v_2 = 0$. Taking up to be the positive y-direction gives:

$$-g = \frac{0 - v_1}{t} \tag{1}$$

The total jump time, including the descent, is $\Delta t = 2t$. The average velocity is given by Eq. 2. This can be rearranged to eliminate v_1 after being substituted into Eq. 1, giving Eq. 3. (h = height):

$$\bar{v} = \frac{h}{t} = \frac{1}{2}(v_1 + v_2) \tag{2}$$

$$-g = -\frac{2h}{t^2} \tag{3}$$

Using the fact that $\Delta t = 2t$, gives Eq. 4: the total airtime during the axel jump.

$$\Delta t = 2\sqrt{2h/g}\,. \tag{4}$$

The same time interval describes the amount of time available to rotate in the air. Eq. 6 describes the relationship between Δt, angular displacement $\Delta\theta$ and angular velocity, ω.

$$\omega = \Delta\theta/\Delta t\,. \tag{5}$$

By substituting Eq. 4 into Eq. 5, the final equation for the angular velocity of the skating jump is:

$$\omega = \frac{1}{2}\sqrt{\frac{g}{2h}}\Delta\theta. \tag{6}$$

Skating jumps have to be landed fully rotated in order to be ratified; jumps with negative grades of execution (GOE) cannot be ratified [8]. For an axel jump of N rotations, the rotations are:

$$\Delta\theta = \left(N + \frac{1}{2}\right)\pi, \tag{7}$$

as the particular jump has an extra half rotation [2]. In the case of a quintuple axel $\Delta\theta = 11\pi$. Assuming that the height is 0.59m, from table 1, inputting this into Eq. 7 results in the angular velocity required to complete a quintuple axel, 49.8 rads^{-1}. Angular momentum, L, the product of angular velocity and I, the moment of inertia is always conserved.

$$L = I\omega = constant,$$
$$I_1\omega_1 = I_2\omega_2. \tag{8}$$

The axel can be modelled in two distinct phases: the start of the jump where the skater spins with both arms and one leg outstretched, resulting in a moment of inertia, I_1, and initial angular velocity, ω_1, and the phase at maximum angular velocity, ω_2, and smaller moment of inertia, I_2. During the second phase of the axel, the skater will have their body tucked in and can be modelled as a cylinder [9].

$$I_1 = I_{body} + I_{arms} + I_{leg}, \tag{9}$$

$$I_2 = \frac{1}{2}MR^2, \tag{10}$$

where M is mass, and R is the radial distance from the skater's centre of mass [9]. Modelling the skater's body as a cylinder and limbs as rods, each moment of

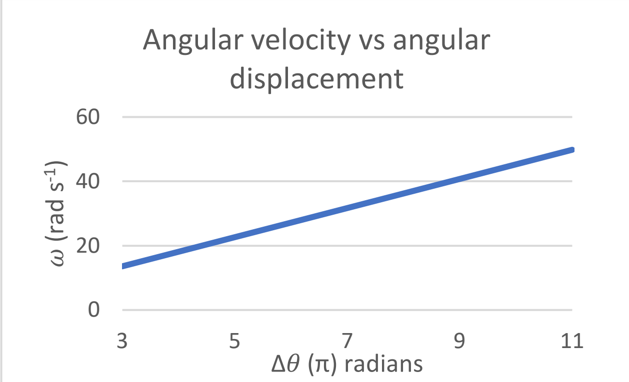

Figure 1 – Graph showing the rotation speed required for a given angular displacement when performing axel jumps, ranging from the 1A to the 5A, using the triple axel data in Appendix A.

inertia that comprises I_1can be described:

$$I_{body} = \frac{1}{2}M_{body}R^2, \tag{11}$$

$$I_{arms} = \frac{1}{12}M_{arms}R_{arms}^2, \tag{12}$$

$$I_{leg} = \frac{1}{3}M_{leg}R_{leg}^2. \tag{13}$$

Assume a skater's arm span to be 1.6 m, one leg to be 0.75 m long and the radius of the 'cylinder' to be 0.1 m, as well as total mass to be 55 kg, both arms will weigh 6.3 kg and one leg 9.2 kg [10]. Substituting these values, as well as the peak rotational velocity ω_2, allows for the calculation of the angular velocity that needs to be generated at the start of the jump:

$$\omega_1 = \frac{I_2}{I_1}\omega_2 = \frac{0.275}{3.530}(49.8) = 3.9\ s^{-1}. \tag{14}$$

Conclusion

This paper has given an overview as to how fast a skater would have to rotate in the air in order to complete a quintuple axel. Assuming that they reach a height of 0.59 m, the average height of a triple axel, a skater would need to rotate at 49.8 rads^{-1} in order to fully rotate a quintuple axel jump and thus ratify it. However, as this is the average height of triple axels, it is likely that a quadruple and quintuple axel would have increased height, and so would require a smaller angular velocity to achieve the full rotations as they would have more airtime. A quintuple axel may be physically possible, however, before quintuple jumps are given base values by the ISU, the long-term injury risk should be seriously considered, and this should be explored in future work.

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Appendix A: 2A and 3A used in table 1.

Skater	Jump	Distance (m)	Height (m)	Speed (km/h)	Notes	Rotation (degrees)
Hongyi Chen	2A	2.10	0.44	14.9		900
Elzbieta Kropa	2A	1.84	0.51	9.7		900
Marina Piredda	2A	2.16	0.40	6.6	Counter entry	900
Julia Sauter	2A	1.73	0.44	12.4		900
Valentina Matos	2A	2.28	0.39	11.3		900
Pernille Sorensen	2A	2.14	0.36	20.2		900
Dasa Grim	2A	2.70	0.37	19.3		900
Eva Lotte Kiibus	2A	2.14	0.39	17.8	Spread eagle entry	900
Alexandra Feigin	2A	2.10	0.34	17.9		900
Natasha McKay	2A	1.80	0.38	5.8		900
Roberta Rodeghiero	2A	1.75	0.33	6.5		900
Anita Ostlund	2A	2.93	0.41	20.5		900
Sophia Schaller	2A	1.99	0.48	9.6	Short entry	900
Aurora Cotop	2A	2.20	0.32	23.4	Tight landing	900
Isadora Williams	2A	1.37	0.32	8.9		900
Kyarha Van Tiel	2A	2.44	0.50	14.3		900
Kailani Crine	2A	1.92	0.43	10.8		900
Yi Christy Leung	2A	2.45	0.38	18.6		900
Emmi Peltonen	2A	2.55	0.50	11.4		900
Loena Hendrickx	2A	1.79	0.39	12.6	Short entry	900
Nicole Rajicova	2A	1.92	0.46	10.9		900
Ivett Toth	2A	1.16	0.39	8.9		900
Alaine Chartrand	2A	2.22	0.40	11.7		900
Eliska Brezinova	2A	2.67	0.48	13.9		900
Gabrielle Daleman	2A	2.22	0.37	11.7		900
Eunsoo Lim	2A	2.40	0.41	15.1	Spread eagle entry	900
Mariah Bell	2A	2.71	0.45	13.5		900
Elizabet Tursynbaeva	2A	2.70	0.40	16.4	Spread eagle entry	900
Laurine Lecavelier	2A	2.38	0.44	15.8		900
Sofia Samodurova	2A	2.29	0.35	16.5		900
Kaori Sakamoto	2A	3.20	0.36	20.7		900
Bradie Tennell	2A	2.59	0.40	15.7		900
Satoko Miyahara	2A	2.18	0.34	13.6		900
Evgenia Medvedeva	2A	2.53	0.35	12.9		900
Alina Zagitova	2A	2.04	0.38	12.5	Counter entry	900
Natasha McKay	2A	2.01	0.39	8.5	Step out	900
Loena Hendrickx	2A	2.01	0.36	13.6		900

Yi Christy Leung	2A	2.50	0.38	17.4		900
Gabrielle Daleman	2A	1.87	0.42	15.7		900
Bradie Tennell	2A	2.84	0.42	18.5		900
Nicole Schott	2A	2.23	0.37	12.6		900
Satoko Miyahara	2A	2.17	0.33	13.0		900
Mariah Bell	2A	2.59	0.42	13.9		900
Eunsoo Lim	2A	2.42	0.36	19.5		900
Kaori Sakamoto	2A	2.47	0.41	15.8	Counter entry	900
Alina Zagitova	2A	2.22	0.41	12.4		900

Table A – Raw data on the double axel jumps, with their height, distance, and landing speeds. Data obtained from the 2019 World Figure Skating Championships [6, 7].

Skater	**Jump**	**Distance (m)**	**Height (m)**	**Speed (km/h)**	**Notes**	**Rotation (degrees)**
Paul Fentz	3A	3.16	0.59	13.9		1260
Julian Zhi Jie Yee	3A	2.21	0.57	12.2		1260
Slavik Hayrapetyan	3A	3.13	0.64	17.9		1260
Alexander Majorov	3A	2.7	0.6	7.2		1260
Vladimir Litvintsev	3A	3.07	0.51	17.2		1260
Andrei Lazukin	3A	2.88	0.63	13		1260
Daniel Samohin	3A	3.22	0.61	13		1260
Burak Demirboga	3A	2.97	0.56	11.6		1260
Luc Maierhofer	2A	2.55	0.38	11		900
Brendan Kerry	3A	2.29	0.59	11		1260
Kevin Aymoz	3A	2.93	0.62	15.7	Counter entrance. Slight lean forward on the landing	1260
Morisi Kvitelashvili	3A	3.51	0.6	17		1260
Michal Brezina	3A	2.73	0.58	21		1260
Keiji Tanaka	3A	2.75	0.6	12.4		1260
Nam Nguyen	3A	2.77	0.49	16.8	Rough landing.	1260
Boyang Jin	3A	2.55	0.57	16	Short entry	1260
Matteo Rizzo	3A	2.59	0.55	16.1		1260
Alexei Bychenko	3A	2.41	0.6	10.6		1260
Vincent Zhou	3A	2.69	0.58	16.7		1260
Alexander Samarin	3A	3.25	0.58	16.1		1260
Yuzuru Hanyu	3A	3.62	0.7	15.3	Back counter entry and twizzle exit	1260
Shoma Uno	3A	3.44	0.51	18.3		1260

Jason Brown	3A	2.35	0.6	14.6		1260
Mikhail Kolyada	3A	2.5	0.65	11.8		1260
Keegan Messing	3A	3.33	0.64	17.3		1260
Nathan Chen	3A	2.66	0.58	17.1		1260
Keiji Tanaka	3A	3.26	0.66	18.9	Hand down	1260
Deniss Vasiljevs	3A	2.89	0.55	22.9	Hand down	1260
Brendan Kerry	3A	2.24	0.6	12.2		1260
Julian Zhi Jie Yee	3A	2.52	0.58	8		1260
Alexander Majorov	3A	2.44	0.55	9.4	3-turn exit	1260
Daniel Samohin	3A	3.14	0.63	13.6	Double footed landing + hand down	1260
Junhwan Cha	3A	2.44	0.61	15.2	Little tight on the landing	1260
Mikhail Kolyada	3A	3.02	0.65	13.8		1260
Morisi Kvitelashvili	3A	3.25	0.59	13.1		1260
Boyang Jin	3A	2.56	0.52	12.9		1260
Michal Brezina	3A	2.17	0.64	12.2		1260
Vincent Zhou	3A	2.78	0.61	12.2		1260
Shoma Uno	3A	3.31	0.52	13.7		1260
Nathan Chen	3A	2.76	0.6	15.7		1260

Table B – Raw data on the triple axel jumps, with their height, distance, and landing speeds. Data obtained from the 2019 World Figure Skating Championships [4, 5].