Supplementary Material

Derivation of tension equation from first principal:

Consider a cable element of incremental length tending to zero (Fig 1.). By resolving forces we can see that the element will experience four forces: the weight of the element [W], the imaginary centripetal force [FC], the tension from the cable above the element [TUP] and the corresponding tension for the cable below [TDOWN].

FC TUP

*Fig 1*

W TDOWN

Since a space elevator is a free standing construction all of the above forces must be in equilibrium the vector equation can be expressed as;

(FU - FD) = FC – W *Eqn 1*

Since the area remains constant in this simple model while the tension is variable the term (FU - FD) can be replaced with AdT. By using this simplification and substituting the explicit terms FC = and W = ma = mω2r we can see that;

AdT = - (Adrρ).ω2r *Eqn 2*

By dividing through by the common factor of Adr we arrive at a differential expression for the tension with respect to the length. It is also at this point it is convenient to make the substitution Rg = ()1/3 where Rg is the radius for geostationary orbit.

= – ρω2r = GMρ *Eqn 3*

= GMρ *Eqn 4*

By integrating equation 4 with respect to the radius from the centre of the earth with limits between R0 (surface of the earth) and Rg (geostationary orbit) we arrive at the expression;

T = GMρ .dr *Eqn 5*

After inserting the boundary condition that T=0 at r=R0, derived from the free standing nature of the cable.

T = GMρ *Eqn 6*

Where the values of the symbols used in the text are;

* G = 6.67x10−11 Nm2/kg2 (Newton’s gravitational constant)
* M = 5.98 x 1024 kg (Mass of earth)
* R0 = 6.37 x 103 m (mean radius of the earth)
* Rg = 42.3 x 106 m (radius of geostationary orbit from the centre of the earth)
* ρ is the density of the cable material in kg/m3
* T is the tension in the cable in Pa

(All values used are from *Physics for scientists and engineers, Tipler and Mosca, 5th edition [2004, W.H Freeman]*)