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# P1_3 Jack and the Breaking Stress

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**Abstract**

We calculate the dimensions of the beanstalk in *Jack and the Beanstalk* required to support the giant castle by approximating its physical properties to that of a red maple tree, and by plotting data for various tree species to estimate its spread. We find a branch length and diameter of (861±4.08)m and (475±2.73)m respectively, and trunk diameter of (1.01±0.0344)km.

## Introduction

In the classic fairy tale *Jack and the Beanstalk*, the protagonist Jack climbs a beanstalk reaching above the clouds, where he finds a giant and a giant-sized castle. In this paper we will calculate the dimensions of this beanstalk by approximating it to a red maple tree due to the large amount of data available for this species. Tree branches have a certain breaking stress indicating the point at which the branch snaps off the trunk. We will use this breaking stress along with estimations for the mass of the castle and data for dimensions of various trees to calculate the minimum diameter of the tree's trunk.

## Theory

The breaking stress $\sigma$ in MPa for a branch of a tree is given by

$$\sigma = \frac{32PL\sin\theta}{\pi d^3} \tag{1}$$

where $P$ is the maximum weight applied to the branch in kN, $L$ is the distance at which the weight is applied from the trunk in m, $\theta$ is the angle made between the branch and the trunk and $d$ is the diameter of the branch in m [1]. To find the diameter we must find $\sigma$, $P$, and $L$ assuming the branch is horizontal making $\sin\theta = 1$. Here we assume that the castle is the full distance $L$ from the trunk.

To find $P$ we note that Jack is the same height as the giant's forearm (ulna) [2]. We compare ulna length to the height of a typical man [3] to give the ratio between the heights of Jack and the giant of $R = 6.52$ and assume this scale factor applies in all three dimensions. The most significant weight on the branch is the castle, so we take an average mass of stone used to build a castle, $m_S = 2.72 \times 10^5$kg [4]. To find the mass of the giant castle we cube $R$ to account for three dimensions and multiply by $m_S$ to give a giant castle mass of $m_G = 7.54 \times 10^7$kg. Thus the weight of the castle, and the maximum weight on the branch, is $P = m_G g = 7.40 \times 10^8$N$=$ $7.40 \times 10^5$kN.

We find the branch length $L$ by plotting known data for the height and spread of a collection of trees, to find a general relationship for various species [5]. This graph is shown in Figure 1, with a least squares fit line showing the approximate correlation. Taking the length of the longest branch, $L$, as half of the spread, the equation for this line is

$$L = (0.542 \pm 0.0189) + (0.344 \pm 0.00163)h \quad (2)$$

for a tree of height $h$. In the story, the beanstalk reaches above the clouds, so taking cloud level as 2km [6] and allowing for a very large branch diameter we substitute height $h = 2.5$km to find a largest branch length of $L = 861$m.

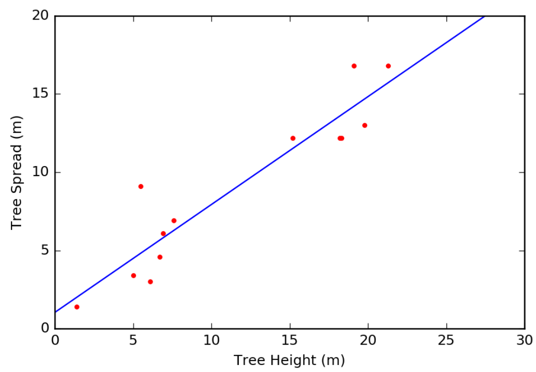


Figure 1: Tree spread against tree height, with least squares fit line plotted in Python

Using a known value for the breaking stress of $\sigma = (60.6 \pm 2.10)$MPa [1], we can rearrange equation (1) for $d$ to find the diameter of the branch, $d = 475$m. The ratio $R_T$ of branch diameter to trunk diameter is also known for the red maple to be $R_T = 0.47 \pm 0.016$ [1]. From this ratio we calculate the trunk diameter for the giant tree to be $d_T = 1010$m$= 1.01$km.

## Discussion

The raw expression for the diameter $d_T$ of the beanstalk's trunk in terms of Jack's height $h_J = 1.76$m, the length of Jack's ulna $u_J = (0.27 \pm 0.0025)$m, the breaking stress $\sigma = (60.6 \pm 2.10)$MPa, the castle mass $m_S = 2.72 \times 10^5$kg, and the ratio of branch and trunk diameters $R_T = 0.47 \pm 0.016$ is

$$d_T = \frac{\left(\frac{32 g m_c h_J^3 L}{\pi u_J^3 \sigma}\right)^{\frac{1}{3}}}{R_T} \quad (3)$$

We use this form to calculate the uncertainties in the diameters of the branch and trunk, using standard error propagation formulae: $\Delta L = 4.08$m, $\Delta d = 2.73$m, and $\Delta d_T = 34.4$m.

One flaw in this analysis is the assumption that the beanstalk can be treated as a red maple. A large amount of data is available for these trees with regards to breaking stress and dimensions, but a more thorough discussion would collect data for trees more similar to a beanstalk.

The diameter of the trunk is reasonable considering the mass the branch must support, but the feasibility of such a tree growing out of small seeds is unclear. These results provide a maximum weight on the branch, so for other objects to be supported the diameters must be treated as minimum values, although most weights are negligible compared to that of the castle.

## Conclusion

We have found the diameter of a tree trunk capable of supporting a giant castle to be $d_T = (1.01 \pm 0.0344)$km, based on data taken from red maple trees. While this is reasonable considering the castle's large weight, an approximation to a beanstalk could be made in the future.

## References


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