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P1_4 The Master Chief Revisited

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Abstract
In this paper we expand upon the findings of "P1_1 The Chief"s Downfall" by examining the trend of the velocity with which the Master Chief impacts the ground at the beginning of the videogame 'Halo 3' with varying initial velocity, using an iterative process, and attempt to find an equation linking the two.

Introduction

In the paper "P1_1 The Chief's Downfall" [1] the final velocity of Master Chief, after he has fallen from a crashing spacecraft, was calculated. In that paper, it was assumed that he fell from a stationary spacecraft, believing that his initial velocity would not affect his terminal velocity. In this paper we calculate the Master Chief's final velocity at varying initial velocities, after noting his initial velocity does have a significant effect on his speed of descent, at large initial velocities. A graph of final velocity against initial velocity was plotted for further analysis and an equation found for the relationship between the two.

Theory

The calculations for the net forces acting upon Master Chief vary little from those performed in [1]. Similarly, we focus on only vertical components of the Master Chief's motion. We find the forces of gravity F_G and drag F_D upon the Master chief by the equations:

$$F_G = mg \tag{1}$$

$$F_D = \frac{1}{2}\rho(z)u(t)^2 C_D A \tag{2}$$

[2] Where $m = 451.3$kg is the mass of the Master Chief, $g = -9.81\text{ms}^{-1}$ is the acceleration due to gravity, $\rho(z)$ is the altitude-dependent density of air discussed below, $u(t)$ is the velocity at time t, $C_D = 1.30$ is the drag coefficient of the Master Chief [1], and $A = 1.19\text{m}^2$ is his approximate face-on area [1]. Resolving these two forces to find a net acceleration upon the Master chief gives:

$$a = \frac{F_G + F_D}{m} = g + \frac{1}{2m}\rho(z){u_t}^2 C_D A \tag{3}$$

Since an iterative process will be used to calculate the velocity of the Master Chief (with a time interval of $\Delta t = 0.001$s) for any starting velocity $u(t - \Delta t)$ we can find the velocity at the next iteration by the equation

$$u(t) = u(t - \Delta t) + a\Delta t \tag{4}$$

The altitude-dependent air density $\rho(z)$ was estimated using the equation:

$$\rho(z) = \rho(0)e^{\frac{-z}{H}} \tag{5}$$

Where $\rho(0) = 1.2250\text{kgm}^{-3}$, the density of air at sea level, and $H = 7.4$km is the scale height. It is taken that the Master Chief's initial altitude of $z(0) = 2000$m does not vary. An iterative process was used to calculate the altitude between each interval ($\Delta t = 0.001$s) with the equation

$$z(t) = z(t - \Delta t) + u\Delta t - 0.5a(\Delta t)^2 \quad (6)$$

Within each iteration, the density was calculated first, followed by the acceleration, velocity, and finally altitude, until the altitude reached $z(t) = 0$.

The above process was repeated for the range of initial velocities, in the range $u_0 = 0\text{ms}^{-1}$ to $u_0 = -15000\text{ms}^{-1}$, with an integer gap between them. This range was chosen based on the fastest velocity attained by a spacecraft falling towards earth [3]. The velocity at zero altitude was found for each initial velocity and plotted on the graph in Figure 1.

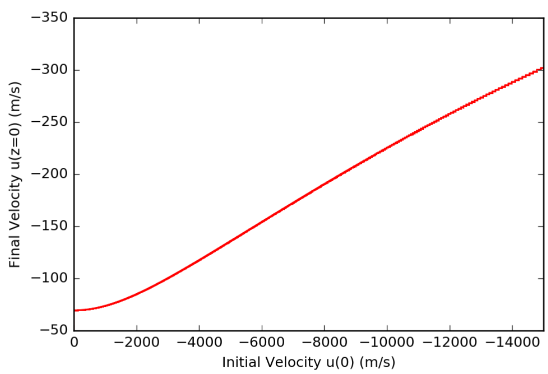

Figure 1: The final velocity of the Master Chief with varying initial velocity. Note that both velocities are negative, as the positive direction is defined as the direction of the altitude.

Discussion

From the trend in Figure 1, it can be seen that, while the terminal velocity dominates at smaller initial velocities creating a low gradient in the trend, at about $u(0) = 2000\text{ms}^{-1}$ the initial velocity dominates over the terminal velocity, creating a more linear trend with a greater gradient.

It is possible to use a least squares fit method to construct a non-linear model of the trend of final velocity as it varies with initial velocity. This was achieved by applying non-linear curve fitting to the 6th order, giving the equation below:

$$\begin{aligned} u(z=0) = & - 4.23 \times 10^{-23} u(0)^6 \\ & - 2.45 \times 10^{-18} u(0)^5 \\ & - 5.86 \times 10^{-14} u(0)^4 \\ & - 7.48 \times 10^{-10} u(0)^3 \\ & - 5.20 \times 10^{-6} u(0)^2 \\ & + 8.42 \times 10^{-5} u(0) - 69.3 \quad (7) \end{aligned}$$

Conclusion

We found in this paper that at velocities of order 10^3ms^{-1} or greater, the initial velocity has a significant effect on the velocity with which the Master Chief impacts the ground, in direct contrast to the assumption made in "P1_1 The Chief's Downfall" [1]. This is modelled in equation (7) above. In order to properly estimate the impact velocity, it is therefore necessary to find a method to estimate the initial vertical velocity of the crashing spacecraft which the Master Chief leaps out of.

References

[1] R. R. Bosley *et al*, *P1_1 The Chief's Downfall*, PST15, (2016).

[2] `www.grc.nasa.gov/www/k-12/airplane/drageq.html`, Accessed: 26/10/2016

[3] `http://stardust.jpl.nasa.gov/cool.html`, Accessed: 26/10/2016