Journal of Physics Special Topics

An undergraduate physics journal

P3_6 Cavorite Pt 2: the Gravity of the Situation

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December 6, 2016

Abstract

We continue to explore the gravitational and atmospheric effects of *Cavorite*, a fictional material that is "opaque to gravitation", by attempting to form a simplified gravitational model of the situation. This gravitational model was found to define the range at which gravitational acceleration will return to near normal as between 0.02 and 12 m. Further research is required to truly model the gravitational acceleration due to Cavorite.

Introduction

In our previous paper *On the Atmospheric Effects of Cavorite* [1], we explored the atmospheric ramifications of placing a circular sheet of Cavorite, a material that is "opaque to gravitation" [2], of radius $r_{cav} = 1$ m. We deduced from looking at how the centre of mass of the Earth z_{cm} and the mass of the Earth M_{eff} seen by a test particle varies with height h above the Cavorite sheet that nearly normal gravitational conditions will be felt at a height of $h \approx 12$m [1]. We therefore concluded that, contrary to what is claimed in the book [2], the Cavorite effect is too localised to cause the atmosphere to vent into space [1].

Building upon the previous paper, we attempt to narrow down the range within which normal g is resumed, with a simplified gravitational model of the situation.

The Model

Due to the change in the "line of sight" of the test particle, both the mass that the particle interacts with, and the effective centre of mass of the Earth (M_{eff} and z_{cm} respectively), will change with h (see Figure (1)). These relationships from the previous paper are given below.

$$M_{eff} = \pi\rho\Bigg(-\left[1 + \frac{r_{cav}^2}{h^2}\right]\frac{h_c^3}{3} + \left[\frac{r_{cav}^2}{h} - R_\oplus\right]h_c^2 - r_{cav}^2 h_c\Bigg) \tag{1}$$

$$M_{eff} z_{cm} = \pi\rho\Bigg(-\left[1 + \frac{r_{cav}^2}{h^2}\right]\frac{h_c^4}{4} - \frac{2}{3}\left[R_\oplus - \frac{r_{cav}^2}{h}\right]h_c^3 - \frac{r_{cav}^2 h_c^2}{2}\Bigg) \tag{2}$$

$$h_c = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \tag{3}$$

where $A = \left[\left(\frac{r_{cav}}{h}\right)^2 + 1\right]$, $B = 2R_\oplus - \frac{2r_{cav}}{h}$, $C = r_{cav}^2$ and ρ is the Earth's density [1].

The gravitational acceleration g_{eff} due to M_{eff} is a complicated problem to solve without the aid of complex computer models, due to the nature of the mass distribution. However, it may be possible to gain an idea of how g_{eff} varies with small values of h if we model the mass distribution as a thin ring (indicated in green in Figure 1), centred at the point on the z axis where $z = z_{cm}$:

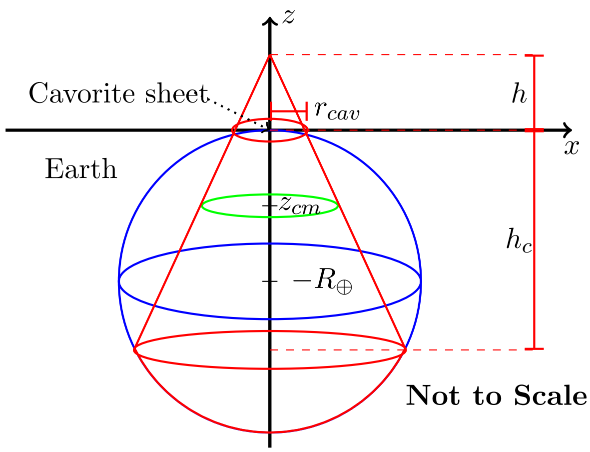

Figure 1: The radius of the green ring used to model M_{eff} is given by $x_{line} = \frac{r_{cav}}{h}(h - z_{cm})$ [1]. The Cavorite is centred on the origin [1].

Using the result for the acceleration due to gravity of a thin ring [3] g_{eff} is given by:

$$g_{eff} = \frac{GM_{eff}(h - z_{cm})}{((h - z_{cm})^2 + x_{line}^2)^{\frac{3}{2}}} \tag{4}$$

This result is plotted below.

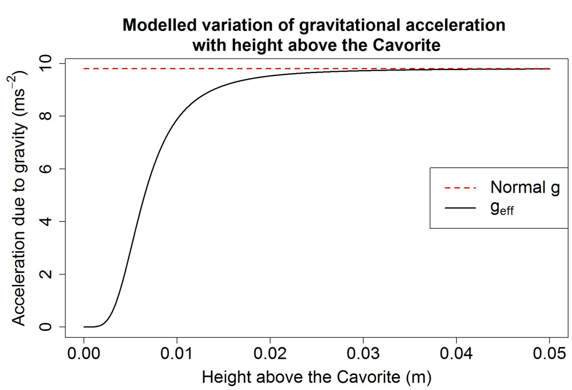

Figure 2: Plot of g_{eff} against h.

Discussion

In Figure (2) g_{eff} tends towards 99% normal gravitational acceleration, g, around $h \approx 0.02$ m, which seems a short distance. This may be because, although the thin ring approximation is reasonable at heights incredibly close to the Cavorite, this approximation breaks down as more of the mass is revealed to the test particle. A truly accurate gravitational model for all h is very difficult without complex computer models, however the range in which normal g is resumed can be narrowed down a little by this model.

We already know from the first paper that a test particle will experience normal g at $h \approx 12$ m [1], but this does not mean that normal g will be resumed exactly at this point. Irrespective of the limitations of the model it does show that the test particle will not experience normal g below $h \approx 0.02$ m. This model also makes the approximation that the Earth has a constant density [1]. If Cavorite was placed above the real Earth, then the lower limit will only increase due to the non-constant density of the Earth. It is unlikely that the upper limit will be affected much by this, as at this upper limit most of the mass of the Earth can be seen by the test particle [1].

Conclusion

We have deduced from a thin ring model that the range at which normal g is resumed is approximately 0.02 m $< h <$ 12 m. More complex methods than are discussed here are required to narrow this range down further.

References

[1] C.J. Middleton, H.W. Buttery, C.D.Y. Moore and R.H. Peck *On the Atmospheric Effects of Cavorite* (PST, Vol. 15).

[2] Wells, H.G. *The First Men in the Moon* (The Modern Library, New York, 2003).

[3] P.A. Tipler and G. Mosca, *Physics For Scientists and Engineers With Modern Physics* (W. H. Freeman and Company, New York, 2008), p. 384.