

P3_3 On the Atmospheric Effects of Cavorite

C.J. Middleton, H.W. Buttery, C.D.Y. Moore and R.H. Peck

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

November 24, 2016

Abstract

We estimate the gravitational and atmospheric effects of a fictional substance known as *Cavorite*, a material proposed by H.G. Wells that is “opaque to gravitation”. We conclude that for a Cavorite sheet of radius 1 m the impact on the gravitational acceleration is only meaningful at less than 12 m above the plate, and is insignificant at heights greater than this. Therefore, the atmosphere is unlikely to vent away as claimed.

Introduction

In the H.G. Wells novel *The First Men in the Moon*, scientist Mr. Cavor postulates (and later manufactures) a substance he calls *Cavorite*, a metal-like sheet which blocks gravity in the same way that lead blocks X rays [1]. When a “thin, wide sheet” of Cavorite is first manufactured, there are extreme atmospheric effects as the air above it becomes weightless [1]. In this paper, we attempt to determine if the atmosphere will vent off into space and asphyxiate the planet, as claimed by Mr. Cavor in the book [1].

We make the following assumptions in our model: the Cavorite effect is taken literally, i.e. the Cavorite completely blocks the gravitational interaction between two masses if placed along their “line of sight” (see Figure 1). The Cavorite sheet radius $r_{cav} = 1$ m (chosen arbitrarily) is insignificant compared to the size of the Earth, thus the Earth is considered to be locally flat, and only the effect along the Cavorite’s axis of symmetry is explored. Also, the Earth is assumed to be a sphere of radius $R_{\oplus} = 6371$ km, mass $M_{\oplus} = 5.97 \times 10^{24}$ kg, and hence constant density $\rho = 5511 \text{ kgm}^{-3}$ [2].

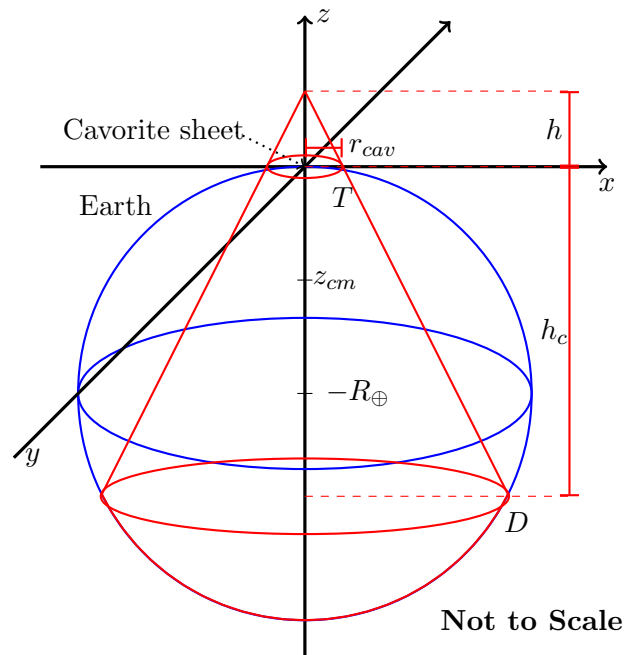


Figure 1: The coordinate system used to model the situation. Any point mass at height h cannot interact with any of the mass of the Earth outlined in red. For the purpose of the model, the Earth has a constant density to enable analytical solutions. The Cavorite sheet is centred at the origin.

Visible Mass

In order to find the mass of the Earth that the point mass can still ‘see’, M_{eff} , the area bounded by the blue curve TD and the red straight line between TD must be rotated around the z axis and multiplied by the density. The equations that describe the full circle and the red line are: $x_{circ} = [R_{\oplus}^2 - (z + R_{\oplus})^2]^{\frac{1}{2}}$ and $x_{line} = \frac{r_{cav}}{h}(h - z)$ respectively. Equating the two lines and solving for z gives us a quadratic equation enabling us to find h_c :

$$h_c = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \quad (1)$$

where $A = \left[\left(\frac{r_{cav}}{h} \right)^2 + 1 \right]$, $B = 2R_{\oplus} - \frac{2r_{cav}}{h}$ and $C = r_{cav}^2$. The value h_c can now be used to define the limits of our mass integral in cylindrical polar coordinates.

$$M_{eff} = \int_0^{2\pi} \int_0^{h_c} \int_{x_{line}}^{x_{circ}} \rho x dx dz d\phi = \pi \rho \left(- \left[1 + \frac{r_{cav}^2}{h^2} \right] \frac{h_c^3}{3} + \left[\frac{r_{cav}}{h} - R_{\oplus} \right] h_c^2 - r_{cav}^2 h_c \right) \quad (2)$$

Centre of Mass

One must recognise that as M_{eff} changes with height h , so must the position of the centre of mass for M_{eff} . The position vector of the centre of mass, \vec{r}_{cm} (measured from the origin), is given by the sum of the products of each mass element m_i and their respective position vectors \vec{r}_{cmi} . Mathematically, $M_{eff} \vec{r}_{cm} = \sum_i m_i \vec{r}_{cmi}$ [2].

Due to the symmetry about the z axis, the x components of $m_i \vec{r}_{cmi}$ will cancel, so $|\vec{r}_{cm}| = z_{cm}$ (the z component of \vec{r}_{cm}) which can be found via:

$$M_{eff} z_{cm} = \int_0^{2\pi} \int_0^{h_c} \int_{x_{line}}^{x_{circ}} \rho x z dx dz d\phi = \pi \rho \left(- \left[1 + \frac{r_{cav}^2}{h^2} \right] \frac{h_c^4}{4} - \frac{2}{3} \left[R_{\oplus} - \frac{r_{cav}}{h} \right] h_c^3 - \frac{r_{cav}^2 h_c^2}{2} \right) \quad (3)$$

Analysis

It is beyond the scope of a single paper to attempt a gravitational model of the situation, and this will be revisited later. However, we can deduce the behaviour of gravity above the plate by analysing how M_{eff} and z_{cm} change with h :

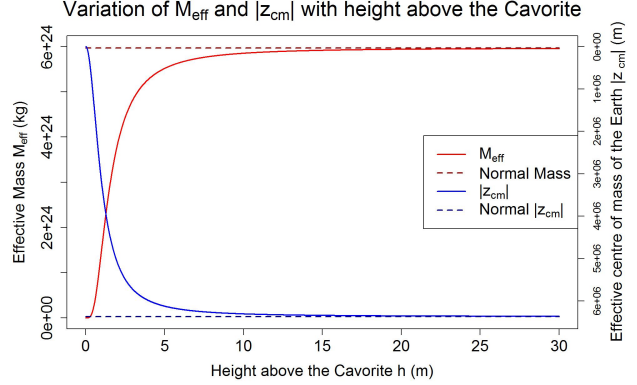


Figure 2: Behaviour of M_{eff} and z_{cm} .

Conclusion

Some of the Earth is always directly below the Cavorite, and so M_{eff} and z_{cm} will never quite reach their nominal values. However, Figure (2) shows that they both quickly tend towards their respective nominal values, and are at 99% of their normal values at $h \approx 12$ m. Hence, it can be logically deduced that a test particle at this height will experience close to normal gravitational acceleration and the Cavorite effect is localised to below 12 m. We conclude that the atmosphere is unlikely to vent away for when $r_{cav} = 1$ m, and the world will not end because of it.

References

- [1] Wells, H.G. *The First Men in the Moon* (The Modern Library, New York, 2003).
- [2] P. A. Tipler and G. Mosca, *Physics For Scientists and Engineers With Modern Physics* (W. H. Freeman and Company, New York, 2008), p. 150.