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P3_8 Elysium: How Fast Did the Atmosphere Escape?

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Abstract

In this paper, the rates at which the atmosphere of the roofless Elysium space-station would escape for various atmospheric angular velocities were considered. The resulting relative mass flow rates were calculated to be between 3.6×10^{-5} - $2.4 \times 10^{-4}\, M\mathrm{s}^{-1}$. From these, the times over which the entire atmospheric mass, M, would be replenished ranged between 1.2 - 7.8 hours, leading to an impractical space-station.

Introduction

It has been previously found that *Elysium*, the unsealed spinning ring-like space station from the 2013 film Elysium [1], would not be able to hold onto its atmosphere, unless the atmosphere were to rotate at a greater rate than the ring (assuming an isothermal atmosphere of temperature T = 300 K, a radius R = 20 km, and a wall height h = 19999 m [2]). In this paper, the relative mass flow rates of the escaping atmosphere from a ring (whose dimensions are those given above), and hence the rates of air supply required to replenish it, are calculated for the atmospheric angular velocities discussed in [2].

Theory

A particle at a height, z, above a rotating ring will experience an acceleration, g, due to its rotation about the ring's centre given by $g = \omega^2(R-z)$ [2], where ω is the particle's angular velocity. Modelling the atmosphere as an ideal gas, and taking it to be in hydrostatic equilibrium (no net force pushing or pulling a gas molecule), it follows that Eq (1) [3] must be obeyed.

$$\frac{d\rho}{dz} = -\frac{\rho m g}{kT} \tag{1}$$

Where ρ is the atmospheric density, k is Boltzmann's constant, T is the atmospheric temperature, and m is the mass of a gas particle. Eq. (1) then finds the atmospheric density profile.

$$\rho = \rho_0 \exp\left[-\frac{g' m}{2kT}\frac{z}{R}(2R - z)\right] \tag{2}$$

Where ρ_0 is the density of the atmosphere at $z = 0$ (given that the pressure is set to 1 atm at this point), and $g' = \omega^2 R$. By integrating Eq. (2) over the volume of the atmosphere bound by height z, the atmosphere's cumulative mass $M(z)$, in terms of the total mass of the atmosphere M, is found.

$$\frac{M(z)}{M} = \frac{1 - \exp\left[-\frac{g' m}{2kTR}\left(2Rz - z^2\right)\right]}{1 - \exp\left[-\frac{g' m}{2kTR}\left(2Rh - h^2\right)\right]} \tag{3}$$

Where h gives the escape height (wall height).

A crude model describing the relative mass flow rate of the escaping atmosphere is now explored. Given that the atmosphere has been described as isothermal, the initial velocities of all escaping particles are equal no matter their respective starting heights z. Since $h - z_{lim}$ will be greater than the distance required to travel for a particle escaping from a height above z_{lim}, it can be assumed that all particles located at $z > z_{lim}$ will escape Elysium over some time τ_{lim}, the time required for a particle starting at z_{lim} to escape. Therefore, the relative mass flow rate for the escaping atmosphere, I_{esc}, can be approximated by:

$$I_{esc} = \frac{1}{6\tau_{lim}} \left(1 - \frac{M(z_{lim})}{M}\right) \quad (4)$$

Where the factor of $\frac{1}{6}$ arises from the assumption that all particles in the atmosphere are travelling in one of six directions (up, down, left, right, forward, backward), and that the same number of particles are travelling in each of the six directions. This assumption was made so as to roughly accommodate for different directions in which the gas particles in the atmosphere move.

Due to the non-constant acceleration from the artificial gravity with changing z; τ_{lim} was found for various atmospheric angular velocities via an iterative process using the equations below:

$$v_{n+1} = v_n - g_n \delta t \ , \quad z_{n+1} = \frac{v_{n+1}^2 + v_n^2}{2g_n} + z_n$$
$$t_{n+1} = t_n + \delta t \quad (5)$$

Where n is the iteration number, v is the velocity of the escaping air particle, t is the cumulative time, and δt is the time increment used. Collisions with other particles were ignored for this process as the mean free path of the rising particles would change with z, due to the reducing ρ. However, it is acknowledged that the inclusion of a mean free path would likely increase the escape time, as collisions would act to reduce the average velocity of a particle.

In [2], the escape temperature, T_{lim}, for a given z, was found via:

$$T_{lim} = \frac{g'm}{3k} \left[2(h - z) - \frac{1}{R}\left(h^2 - z^2\right)\right] \quad (6)$$

Solving Eq. (6) for z gives z_{lim}, the height above which all the atmosphere can escape.

Discussion

The iterative process described by Eq. (5) was performed until $z_n = h$, at which point $t_n = \tau_{lim}$. This was performed for each ω discussed in [2], where their respective initial heights, z_{lim}, were found via Eq. (6). The initial heights, z_1, were actually 1 cm above z_{lim} to avoid long iterations (it was assumed that the resulting difference in escaping mass was negligible). Values of I_{esc} (given in Table (1)) were then found by inserting the results for τ_{lim} and Eq. (3) into Eq. (4).

ω (rads^{-1})	z_{lim} (km)	Escaping Mass (M)	I_{esc} (Ms^{-1})	T (hrs)
0.031	3.78	0.066	2.4×10^{-4}	1.2
0.038	6.76	0.020	9.6×10^{-5}	2.9
0.044	8.53	0.006	3.6×10^{-5}	7.8

Table 1: I_{esc} for different ω, and their respective z_{lim}, $\frac{1}{6}\left(1 - \frac{M(z_{lim})}{M}\right)$, and T values.

For a steady state system in which the atmosphere is constantly replenished, Table (1) gives $T = \frac{1}{I_{esc}}$, the time over which an entire atmosphere is replenished.

Conclusion

Elysium's atmosphere was found to escape at rates between 3.6×10^{-5} - 2.4×10^{-4} Ms^{-1}. To remain in a steady state, an entire atmosphere would need replacement over the course of 1.2 - 7.8 hours. Coupled with the high wind speeds due to the angular velocities (as discussed in [2]), it can be concluded that the designers' insistence on omitting a roof makes the station uncomfortable to inhabit and impractical to maintain.

References

[1] http://tinyurl.com/7bod7t6 (18/10/16)

[2] H. Buttery et al. *P3_4 Elysium: Where'd the Atmosphere Go?*, PST 15, (2016)

[3] J. R. Holton, *An Introduction to Dynamic Meteorology Fourth Edition* (Elsevier Academic Press, London, 2004)