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P5_5 Drag on Dragons

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Abstract

By comparison of the exertion needed to overcome lift and drag forces of two dragons, one fat with a supplementary hydrogen source and one thin without, we have calculated the turning point velocity in which it is more beneficial to be either size. The wind velocity, or turning point velocity, was found to be 14.9 ms^{-1}; below this value the fat dragon will be more advantageous and, conversely above, the thin dragon.

Introduction

Dragons are mythical creatures found in both popular fiction stories and fantasy films, such as the primary antagonist Smaug, in J.R.R Tolkien's The Hobbit novel. Dragons are renowned for their size, fire breathing capabilities and their ability to fly. In this paper we explore the possibilities of dragon flight from a physical standpoint, with an emphasis on the physical dimensions that would permit such a creature to fly; we examine whether a larger dragon, with an extra lift capability provided by a stomach full of elemental hydrogen, would prove more suitable for flight than a thinner dragon with no such hydrogen store. A treatment of the drag forces that may act in these two cases is done, and compared to the other forces that may act upon such an aerial body.

Theory

To find whether it is better to be a thin dragon, or a fat dragon filled with hydrogen, we compare the work done for the drag and lift of each dragon, which we term 'effort'. We can assume that the distances are the same for each dragon (unit distance) as they follow the same path ; therefore, we look at the force as the only variable. (It is also worth noting that we are only looking at the magnitudes of the forces, rather than their vectors.) By comparing the differences in the lift forces and the drag forces, separately, acting on the two dragon sizes, we can find the windspeed where the effort required to overcome increased drag forces due to being a fat dragon outweighs the added lift due to the hydrogen.

We model the dragon's shape on long cylinders to find their coefficient of drag. We base the proportions for the dragons on Smaug and assume that both dragons should have a length of 20 m, with the thin dragon having a diameter of 1 m and the thick dragon a diameter of 3 m - both diameters were used to find the cross sectional, or face, area A_f. We also assume that the wings are perfect aerofoils of area, $A_w = 120$ m^2.

A graph is produced, Figure 1, which compares the lift and drag forces on the dragons with the wind velocity (squared) experienced by the dragons. The larger dragon with the hydrogen store will have a greater lift, due to the buoyancy force provided by the lighter-than-air hydrogen, but will have a larger drag due to the greater surface area. The smaller dragon will not have this extra buoyancy force, but will be more aerodynamic due to the smaller surface area. We find at what wind speed it is more beneficial to be a smaller dragon. In both cases, the weight of the wings are neglected, as they are assumed to be identical in the two cases and a difference between the two is taken, negating any adverse effects they may have.

Equation 1 [1] gives the force of lift for the dragon, the value of which is dependent upon a number of variables: the path difference, m, a dimensionless value which is 1.1 for a perfect aerofoil; ρ_A, the air density equal to 1.225 kgm^{-3}; and v, the wind speed.

$$F_L = (m^2 - 1)\frac{v^2 A_w \rho_A}{2} \quad (1)$$

The fat dragon has an additional lift force due to the hydrogen layer surrounding it. The hydrogen produces a buoyancy force that is supplements the lift force, given by [2]

$$F_H = (\rho_A - \rho_H)V_H g - m_H g \quad (2)$$

where ρ_H is the density of hydrogen [3], 0.08988 kgm^{-3}, V_H is the volume of hydrogen outside the core of the dragon (this will be the same size and density as the thin dragon) 126 m^3, m_H is the mass of hydrogen and g is the acceleration due to gravity; the final term, $m_H g$, can be ignored due to the negligible hydrogen mass compared to the dragon's mass.

Equation 3 [4] is the force of drag on the dragon, where C is the dimensionless coefficient of drag, 0.82 for a long cylinder [4], ρ_A is the density of the air, again 1.225 kgm^{-3}, A_f is the area of the face of the dragons, 7.1 m^2 and 0.7 m^2 and v is again the wind speed.

$$F_D = CA_f \rho_A v^2 \quad (3)$$

The velocity at the point at which the difference between the lift force on the thin dragon, F_{L1}, and the larger dragon, F_{L2}, is equal to the difference between the drag force on the thin dragon, F_{D1}, and the larger dragon, F_{D2}, may now be found.

$$|F_{L1} - F_{L2}| = |F_{D1} - F_{D2}| \quad (4)$$

It should be noted that the difference between the two dragon's lift force is equal to the lift force due to hydrogen (F_{LH}). Therefore by knowing the difference in lift forces, and using the equation for drag (Equation 3) we can rearrange for the turning point velocity.

$$v = \sqrt{\frac{F_{LH}}{C\rho_A(A_{f1} - A_{f2})}} \quad (5)$$

This value of the wind speed provides the turing point, where above v it favours one dragon compared to the other, and below v it will be vice versa.

Results

The value of v found using Equation (5) was calculated to be 14.9 ms^{-1} (221.7 m^{2}s^{-2}).

Figure 1 can be used to graphically solve for v in Equation (5). The turing point velocity is when the difference in drag forces (green and black lines) equals the difference in lift forces (red and blue lines) for each dragon.

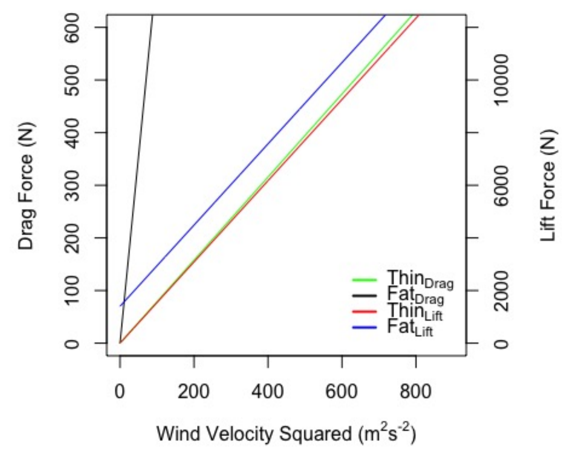

Figure 1: Drag force from Equation (1) (green and black) and lift force from Equation (2) (red and blue) against wind velocity squared.

Conclusion

In conclusion, we have found that a fat dragon would prove more beneficial for a wind speed less than 14.9 ms^{-1}, and above this value the thin dragon is more advantageous. It may be noted that the thin dragon would still require a source of propellant to maintain its fire breathing abilities, and should this be the same quantity of hydrogen as the larger dragon, however greatly compressed, further consideration of the lift force would be required. Furthermore, a combination of the two scenarios, of the thin dragon and fat dragon with a hydrogen source, would likely be the most effective. The authors recommend staying away from such creatures should you ever find one, given their likely somewhat volatile personality and fire breathing abilities.

References

[1] `https://goo.gl/8Z9wTP` [Accessed 07/11/16]

[2] `https://goo.gl/X6Up8N` [Accessed 08/11/16]

[3] `https://goo.gl/fIkuI8` [Accessed 08/11/16]

[4] `https://goo.gl/MaS3oK` [Accessed 07/11/16]