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P2_5 Jovian Gravitational Perturbations on Earth's Orbit

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Abstract

In a hypothetical situation in which the solar system contains varied masses of Jupiter, the orbit of the Earth may fall subject to considerable gravitational perturbations. With the aid of the Newtonian gravity simulator, Universe Sandbox^{TM}, the effects of these perturbations can be observed. The orbit is found alter by an approximate 30x10^6 km before being ejected from the solar system, at a Jovian mass of $0.7M_\odot$.

Introduction

The Earth orbits the Sun at an average radius of 1 AU, with limited gravitational perturbations from the surrounding planets. This is possible due to the lack of mass in the neighbouring bodies, which disallow significant gravitational influences. In this paper, we will consider the hypothetical situation in which the aforementioned is false. By adding considerable alterations to the mass of Jupiter, we investigate the observable perturbations acting on the orbit of the Earth.

Model

In order to test the conceptual foundation of the investigation, we treat the Sun-Earth-Jupiter system as an approximate circular-orbit restricted three-body problem. We assume that the mass of the Earth is negligible compared to the other two bodies and that the Sun and Jupiter move in circular orbits about their centre of mass. In addition, we assume that orbital motion occurs only in the orbital plane of the Sun and Jupiter [1].

Additional consideration of the the center of mass, or barycentre, of the system is required as the mass of Jupiter, M_J, is increased. This is due to the inward migration of the Jovian orbit from this change. The magnitude of the migration is granted by, Equation (1) [2].

$$r_1 = \frac{M_J}{M_\odot + M_J}\, a \tag{1}$$

Where r_1 is the distance from the centre of the Sun to the barycentre, a is the distance between the centres of the Sun and Jupiter, and $M_\odot$ is the solar mass. In the initial case, where $M_\odot$ and M_J hold their usual values and a is the semimajor axis of Jupiter's orbit, r_1 is calculated to be 742,000 km and the Sun orbits a point just above its surface.

Simulation

For the investigation, the N-body Newtonian-gravity simulation software, Universe Sandbox^{TM} [3] is used to simulate the system described by the model. By altering the Jovian mass, the observable effects on the orbital distance from the Earth can be monitored. As the mass is increased to 0.1 $M_\odot$, the effects of the gravitational perturbations on the

Earth orbit become observable. As such, the simulation should be observed over the period of single orbit to gauge the variation in the distance of the orbit from its averaged value.

By treating the Earth as a minor-body, the application of Cowell's Formulation may be applied to gain a further understanding into the causation of the variations observed (See *"Fundamentals of Astrodynamics and Applications"* [1]).

$$\ddot{\mathbf{r}}_{\mathbf{i}} = \sum_{j=1_{j\neq i}}^{n} \frac{Gm_j(\mathbf{r_j} - \mathbf{r_i})}{r_{ij}^3} \quad (2)$$

Figure 1 exhibits the simulation resultants up to the a point in which the Earth is ejected due to the an increase in the barycentre proximity relative to the orbit of the orbit.

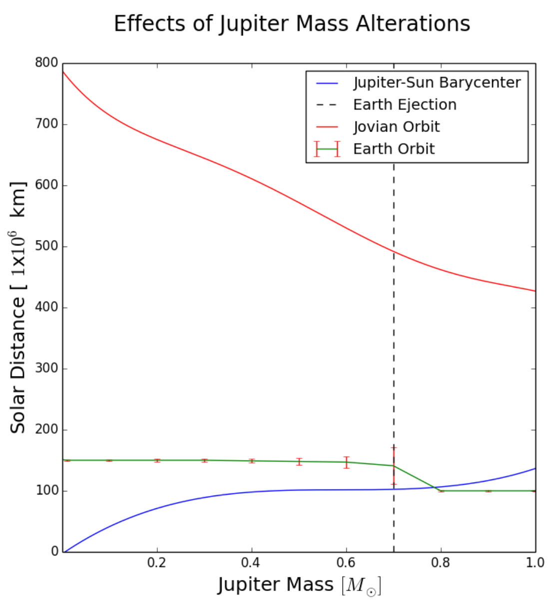

Figure 1: The observable effects of the alterations caused by Jovian mass alterations on the Jupiter-Solar barycentre, Earth orbital distance and the distance of the Jovian orbit.

Discussion

From Figure 1, we can see a clear change in the variation of the orbital distance from the Sun for the Earth. This appears to become apparent as the Jovian orbit draws closer to the Sun for larger Jovian masses. As the mass of Jupiter approaches 1 $M_\odot$, the barycentre exists at a distance greater than the standard orbital distance of the Earth (>1 AU).

According to Kopparapu et al.(2013), the habitable zone around the Sun exists at 0.99-1.69 AU [4]. Therefore, Orbital variations outside of this zone may also render the Earth inhabitable, this may occur at 0.7 $M_\odot$ (as suggested by Figure 1).

Conclusion

It is apparent that alterations made to the mass of Jupiter have a considerable effect on the orbit of the Earth. Whilst variations are small in comparison to the overall scale of the considered systems, the effects of a shorter barycenter and increased Jovian mass, lead to the overall ejection of the planet. Variations caused by the Jovian perturbations may also lead the Earth out of the habitable zone, making the planet inhospitable.

References

[1] Vallado, D. A., *Fundamentals of astrodynamics and applications*, volume 12, Springer Science & Business Media, 2001.

[2] Tipler, P. A. and Mosca, G., *Physics for scientists and engineers*, Macmillan, 2007.

[3] Giant-Army, Universe-Sandbox^{TM}, 2015.

[4] Kopparapu, R. K., The Astrophysical Journal Letters **767** (2013) L8.