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A2_3 Harry Potter and the Concussion at Platform Nine and Three Quarters

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Abstract

We investigate the likelihood of Harry quantum tunnelling through a wall to reach platform $9\frac{3}{4}$ by modelling him as a giant free particle with the mass and velocity of a small boy. We find that the probabilities associated with this type of tunnelling is $10^{-2\times10^{45}}$ for a speed of $7ms^{-1}$ and $10^{-1\times10^{45}}$ for 0.998c.

Introduction and Theory

In the franchise Harry Potter, students of Hogwarts reach platform $9\frac{3}{4}$ by running through a wall. In this paper we will calculate the possibility of quantum tunnelling through the wall and thus reaching the platform without magic.

Quantum tunnelling is an observed phenomenon in quantum mechanics where particles can overcome energy barriers greater than their own energies when classically they would be reflected. In the case of Harry Potter running at the wall we assume the energy barrier he must overcome is the mass energy of the wall, i.e. $E = mc^2$. We take the wall dimensions to be 4 $\times$ 2 $\times$ 0.1m which gives a wall volume of $0.8m^3$. We assume the wall is made of concrete which has a density of 2400kgm^{-3} [1]. This allows us to calculate the mass of the wall to be 1920kg. This is equivalent to an energy of 1.728×10^{20}J. Using $60kg$ as the mass of Harry and assuming he runs at the wall at roughly 7ms^{-1} we can calculate the energy of Harry's wavefunction using

$$E = \frac{\hbar^2 k^2}{2m}, \tag{1}$$

where m is the particle mass and k is the wavenumber given by

$$k = \frac{2\pi}{\lambda} \tag{2}$$

where λ is Harry's wavelength. This model assumes that Harry is one giant free particle, thus by taking Harry's de Broglie wavelength

$$\lambda = \frac{h}{p} \tag{3}$$

and using Eq's. (1) and (2) we calculate that Harry has an incident energy of 1470J at the wall. Eq (3) gives the De Broglie wavelength in terms of h, Planck's constant and p, the particle's momentum.

To calculate the probability of Harry tunnelling through the wall we must consider the amplitude of the incident and transmitted waves which are related via Eq. (4) [2]

$$\frac{|F|^2}{|A|^2} = \frac{16E(V-E)}{V^2} exp(-4\kappa) \tag{4}$$

Where E is the particle energy (in this case Harry), V is the potential barrier energy,

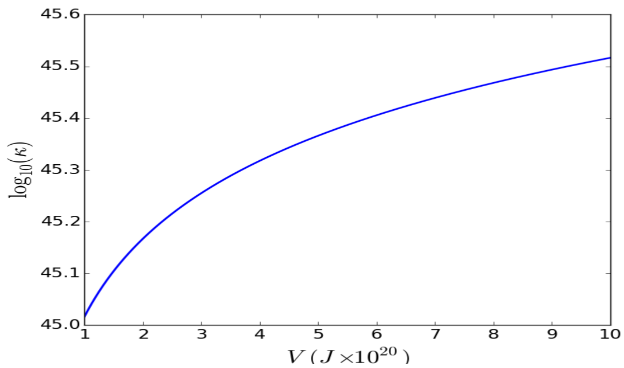

Figure 1: The graph shows how the logarithm of the wavenumber in the potential barrier changes for increasing potential barrier strengths in the case of Harry Potter running at a wall. The graph shows that, were the wall larger than assumed it could lead to a significant change in the value of κ.

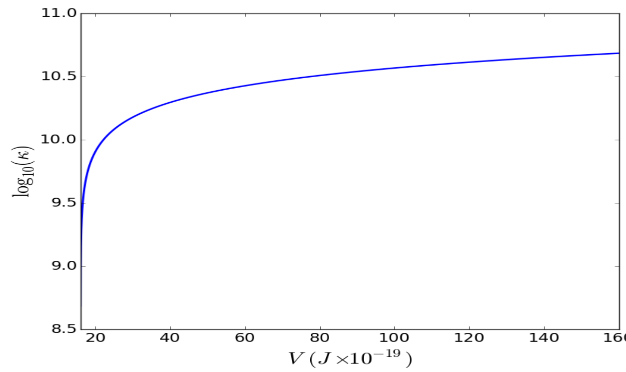

Figure 2: The logarithm of the wavenumber in the potential barrier is compared to varying potential barrier strengths in the case of an electron incident upon potential barrier strengths one might experience in nature i.e. where we would expect to see quantum tunnelling occur. An electron of energy 10eV was used.

$|F|^2/|A|^2$ is the probability of tunnelling and κ is the wavenumber within the barrier which is given by Eq. (5) [2]

$$\kappa = \frac{[2m(V-E)]^{1/2}}{\hbar} \quad (5)$$

Where $\hbar$ is the reduced Planck's constant. By taking the difference in the log scales used in Figures (1) and (2), we can see that κ is of order 10^{35} times larger for the case of a human tunnelling through a wall than an electron tunnelling through a potential barrier where we would expect to observe quantum tunnelling occurring in nature.

Results

Using Equation (5) and all the parameters discussed we find that Harry's wavenumber within the potential barrier is 1.36×10^{45}. As this value is so large, we can assume that the exponential term dominates in Equation (4) and thus it becomes

$$P = \frac{|F|^2}{|A|^2} = exp(-4\kappa) \quad (6)$$

so the probability of Harry successfully tunnelling through the wall to platform $9\frac{3}{4}$ is calculated to be $10^{-2\times10^{45}}$. To calculate this we take the natural logarithm of the probability then convert this to a base-10 logarithm via the equation

$$log_{10}(P) = \frac{ln(P)}{ln(10)} \quad (7)$$

Taking the inverse of this then yields an answer in the form of 10^P.

If, to counteract this low probability, Harry were to run at the relativistic speed of 0.998c, he would increase the probability of tunnelling through the wall to $10^{-1\times10^{45}}$. In this case relativistic momentum must be considered where $p = \gamma mv$, here γ is the relativistic Lorentz factor.

Conclusion

We find that the probability of Harry tunnelling through the wall without magic is incredibly unlikely due to the sheer size of the potential barrier. Even travelling at very high relativistic speeds is not enough for him to overcome this obstacle.

References

[1] http://www.engineeringtoolbox.com/concrete-properties-d_1223.html accessed on 05/11/16

[2] Alistair I.M Rae *Quantum Mechanincs* pp25-26 and pp30-31 (Taylor & Francis, 2008)