A2_8 Propeller Hats Do Not Make Good Parachutes

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November 23, 2016

Abstract

In this paper we investigate whether a person can survive a fall using only a propeller hat to save them as often depicted in cartoons. We find the propeller requires an angular velocity of 58 rad.s^{-1} for a person to land at a safe velocity of 7 ms^{-1}.

Introduction

Propeller hats are often seen within cartoons and games as a form of transport, such as Inspector Gadget's flying propeller hat and Mario's propeller mushroom. We can assume for these examples, that the propellers have a form of engine to produce the spin, which creates enough lift to allow someone to be raised upwards. However, in this paper we calculate how fast a propeller hat with no internal power, driven solely by drag, needs to spin so it can be used to slow a falling persons descent enough for them to survive. This method of controlled falling is seen in nature with maple seeds (commonly known as helicopter leaves) as well as in man-made objects such as helicopters. Helicopter pilots are taught to use this technique known as autorotation in the case of an engine failure.

Theory

At a given height, the rate at which you fall is determined by how potential energy (PE) is converted into kinetic energy (KE). Assuming a lossless system, where no energy is converted to heat or sound, an object of mass M that begins to fall from height h to the ground will have all of its PE converted to KE. Drag can be ignored as long as h is sufficiently small, so that a terminal velocity is not reached. The acceleration experienced is g (9.81 ms^{-2}) and is directed towards the Earth. Therefore, the total energy, given by Equation (1), is the sum of KE and PE which remains constant (C).

$$\frac{Mv^2}{2} + Mgz = C \qquad (1)$$

Where z is the height above the ground and v is the velocity at z.

Considering the same object but with a propeller attached, we find some of the PE is now converted into the rotational kinetic energy of the propeller. This is driven by drag. We assume the presence of the propeller means that drag is no longer negligable, due to a larger area, and the kinetic energy lost to drag is converted back into the rotational kinetic energy of the propeller. So whilst drag slows the falling person down, the rotating propeller means the total energy of the system is unchanged. As a consequence of conservation of energy, we can infer from Equation (2) that for the same total energy as Equation (1) (starting at the same h), the object with a propeller must have a slower velocity u. [1]

$$\frac{Mu^2}{2} + \frac{I\omega^2}{2} + Mgz = C \qquad (2)$$

Where I is the propellers moment of inertia and ω is the angular velocity.

Using Equation's (1) and (2) we can find the required ω to give a velocity u that would have otherwise been a fatal velocity v without the propeller.

We consider two people, one without a propeller (person 1) and one with a propeller (person 2). They have the same total mass (including a propeller for person two, and dead weight of equivalent mass for person one) falling from a height h. As they start falling person 2 miraculously unfolds a propeller hat of mass m and puts it on in an attempt to slow down.

Equation (3) describes their respective total energies, where the left hand side is for person 1, the right hand side is for person 2.

$$\frac{1}{2}Mv^2 + Mgz_1 = \frac{1}{2}Mu^2 + Mgz_2 + \frac{1}{2}I\omega^2 \quad (3)$$

Where z_1 and z_2 are the respective heights of persons 1 and 2 above the ground for their given respective velocities v and u. By taking the energy at the instant at which person 1 lands, and the energy at the instant at which person 2 lands shortly after, their heights will be $z_1 = z_2 = 0$. Using this to simplify Equation (3) and rearranging for ω gives

$$\omega = (\frac{M}{I}(v^2 - u^2))^{1/2} \qquad (4)$$

Which is the required angular velocity of the propeller to save person 2.

The velocity of person 1 v can be calculated by equating initial PE and final KE, this gives

$$v = \sqrt{2gh} \qquad (5)$$

again assuming negligible drag without the propeller for small h.

I is given by Equation (6) [1]

$$I = \frac{m}{12}(r^2 + c^2) \qquad (6)$$

Where r and c are the length and width of the propeller respectively, assuming the propeller is rectangular. So using Equation (4) and estimating a safe velocity at which a person can land u, the angular velocity of the propeller required to save person 2 can be found.

Results

We assumed person 1 and 2 to have mass $M = 70 + m$ kg. Where m is the mass of propeller for person 2 and the mass of the dead weight for person 1. The propeller has length r, width c and thickness t of 2 m, 0.1 m and 0.02 m respectively. It is made from alluminium with density 2700 kgm^{-3} [2] and hence has mass $m = 10.4$ kg. We assumed that a safe speed for our person to land is $u = 7$ ms^{-1} (around 15 mph)[3]. For a fall of $h = 10$ m, the required angular velocity was calculated as 58 rad.s^{-1} which is 553 rpm (rotations per minute).

Discussion

In our scenario we assumed a frictionless system, laminar flow and no thermal transfers of energy. As well as assuming all drag force on the propeller is converted into angular velocity of the propeller. The angular velocity we found means our propeller is performing roughly 9 rotations per second so that it can land at a safe speed. This is very fast, suggesting that the drag will not be a large enough driving force.

References

[1] Tipler, P.A. and Mosca, G., Physics For Scientists and Engineers, (W. H. Freeman, New York, Sixth Edition, 2008), p293, p295

[2] http://www.engineeringtoolbox.com/density-solids-d_1265.html

[3] http://www.forbes.com/sites/tanyamohn/2011/09/30/low-driving-speed-can-cause-serious-pedestrian-injury-and-death-report-finds/#1b2312c0598b