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A3_5 Earth's Radioactive diet

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Abstract
In this paper we consider the mass loss of Earth due to radioactive decay of Uranium, Thorium and Potassium. We do this by considering the mass loss during the decay of each of the isotopes. We found that the mass loss of the Earth would be 4.32 $\times$ 10^{17} kg by the time all nuclear fission reactions have occurred for these isotopes. This results in a total energy output of 3.88 $\times$ 10^{34} J.

Introduction

On Earth there are many radioactive isotopes of chemical elements that decay naturally and as a result they convert mass into energy. This process is responsible for internally heating the Earth and is also used as an energy source in nuclear fission reactors. Some of the isotopes that contribute the most to this process are Uranium-235, Uranium-238, Thorium-232 and Potassium-40 and as a result of their decay reaction they produce around 47 gigawatts of power on Earth at their current rate of decay [1].

Theory

The reactions that are responsible for the majority of the decays being considered are:

$$^{40}_{19}K \rightarrow^{40}_{20} Ca + \beta^- + \gamma \tag{1}$$

$$^{232}_{90}Th \rightarrow^{228}_{88} Ra +^4_2 \alpha + \gamma \tag{2}$$

$$^{235}_{92}U \rightarrow^{231}_{90} Th +^4_2 \alpha + \gamma \tag{3}$$

$$^{238}_{92}U \rightarrow^{234}_{90} Th +^4_2 \alpha + \gamma \tag{4}$$

where γ is the energy released from the decay [2][3]. The mass loss for each of the reaction can be found by using Eq (5)

$$\Delta m = (m_R - m_p)m_n \tag{5}$$

where Δm is the mass lost in the reaction, m_R is the sum of the mass of the reactants, m_p is the sum of the mass of the products of the decay and m_n is the mass of a neutron (this is used to convert from atomic mass units (amu) to kg). The energy released from these reactions can be found by using Eq (6)

$$\Delta E = \Delta mc^2 \tag{6}$$

where ΔE is the energy released in the decay reaction and c is the speed of light

Using the atomic masses in [4], [5](Thorium isotopes) and [6] (Radium isotope) along with Eq (5) and Eq (6) we found the mass and energy loss per reaction for each of the isotopes

To find the inital total mass of each isotope on Earth, the abundancies [7] and half lives of the isotopes were found [8]. The total mass was found by multiplying the abundancy by the mass of Earth. N_0 was found by using Eq (7):

Reaction	Mass loss (kg)	Energy loss (MeV)	$N(t)$ (kg)	N_0 (kg)	Total mass loss (kg)	Total energy ouput (J)
$^{40}_{19}$K (1)	1.44×10^{-30}	0.81	1.66×10^{21}	2.00×10^{22}	4.32×10^{17}	3.88×10^{34}
$^{232}_{90}$Th (2)	7.32×10^{-30}	4.12	6.30×10^{17}	7.88×10^{17}	1.49×10^{13}	1.34×10^{30}
$^{235}_{92}$U (3)	8.39×10^{-30}	4.72	1.39×10^{15}	1.32×10^{17}	2.82×10^{12}	2.54×10^{29}
$^{238}_{92}$U (4)	1.93×10^{-29}	10.9	1.98×10^{17}	4.02×10^{17}	1.95×10^{13}	1.76×10^{30}

Table 1: This table shows the mass loss and energy loss for each of the initial starting points of the decays in Eq (1) - Eq (4). It also shows the current and initial masses of each of the starting isotopes as well as the total mass and energy loss due to the decay of all of each isotope on Earth.

$$N_0 = N(t)\frac{1}{2}^{\frac{-t}{\lambda}} \tag{7}$$

where N_0 is the initial total mass of an isotope on Earth, $N(t)$ is the current total mass of an isotope on Earth, t is the time since Earth formed (4.6×10^9 years) and λ is the half life of the isotope.

The parts per million of Uranium was found by finding the total ppm of Uranium [7] and then using the ratio of $^{235}_{92}$U:$^{238}_{92}$U. The ratio was found to be 0.7% [9].

From the data in Table (1) and using Eq (8) we can find the total mass loss due to the decay of every nucleus for all of the isotopes considered.

$$\Delta m_{tot} = \frac{m_{N_0}}{m_R m_n}\Delta m \tag{8}$$

where m_{N_0} is the total initial mass of the isotope on Earth. Then using Eq (6) and changing Δm to Δm_{tot} we can also find the total energy output of all of the nuclei.

Discussion

In order to calculate the mass and energy loss of the Earth we made the following assumprions. First we assumed that all other isotopes of the elements mentioned were negligible. We also only considered the first reaction in the decay chains. Finally we also only considered the most common decay products.

Conclusion

In this paper we calculated that the total mass loss due to the decay of the entire amount of the isotopes of $^{40}_{19}$K, $^{232}_{90}$Th, $^{235}_{92}$U and $^{238}_{92}$U on Earth was around 4.32×10^{17} kg. The total energy output of this is 3.88×10^{34} J. From the calculations made we have shown that this is roughly the same mass and energy output of the potassium - 40 isotope. This suggests that this isotope is much more influential to the results than the other 3 isotopes considered in this paper.

Further work could include considering other radioisotopes on Earth. Another area that could be explored in future work could be to follow through on one or more of the decay chains.

References

[1] `goo.gl/tniADZ` accessed on 23/10/2016

[2] `goo.gl/YYpzKC` accessed on 30/10/2016

[3] `goo.gl/iNqv1e` accessed on 07/10/2016

[4] `goo.gl/SDpOSN` accessed on 30/10/2016

[5] `goo.gl/GSwY2b` accessed on 30/10/2016

[6] `goo.gl/Ne81Jz` accessed on 30/10/2016

[7] `goo.gl/jiVNJz`, page 4 accessed on 30/10/2016

[8] `goo.gl/1ONpnD` accessed on 07/11/2016

[9] `goo.gl/nkpFHL` accessed on 30/10/2016