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A6_8 Supermassive Black Holes Merger: Violation of the Second Law of Thermodynamics?

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Abstract

The future merger of supermassive black holes residing in the centre of the Milky Way and Andromeda is investigated. Using the Bekenstein-Hawking formula, it is found that the second law of thermodynamics is not violated as the entropy of the system increases by 4.8%.

Introduction

In about 4 billion years, the Milky Way and Andromeda galaxies will collide in a spectacular event [1]. Even with a combined total of 1.3 billion stars, the chance of one colliding with another is negligible [1]. On the other hand, the supermassive black holes residing in the centre of the galaxies will come into "contact" and merge. At a distance of one light year, the two will start to orbit each other and radiate gravitational waves [1]. These waves are ripples in the curvature of space-time and were first detected in 2016 using the Advanced LIGO detectors [2].

We investigate if the second law of thermodynamics, which states that the entropy of an isolated system increases or remains constant over time will be violated in the merger event.

Theory

The power radiated as gravitational waves due to the two supermassive black holes orbiting each other is given by [1]:

$$\frac{dE}{dt} = -\frac{32}{5}\frac{G^4}{c^5}\frac{(m_1 m_2)^2(m_1+m_2)}{r^5}, \quad (1)$$

where G is the gravitational constant, c is the speed of light in a vacuum, r is their separation distance and m_1 and m_2 are the masses of the black holes respectively.

As they radiate, their orbit will start to decay over time and the rate at which this occurs is approximated by [2]:

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{(m_1 m_2)(m_1+m_2)}{r^3} \quad (2)$$

Rearranging equation 2 for dt and substituting into equation 1 leads to:

$$dE = \frac{1}{2}\frac{Gm_1 m_2}{r^2}dr \quad (3)$$

Intergrating from the final seperation, r_f, to the initial seperation distance, r_i, gives the total energy emitted during the inspiral.

$$E = \frac{1}{2}Gm_1 m_2(\frac{1}{r_f} - \frac{1}{r_i}) \quad (4)$$

Assuming energy is conserved and that the mass loss of the binary system is only due to the emission of gravitational waves, the mass, M, of the black hole formed after merger is equal to:

$$M = m_1 + m_2 - m_{\text{rad}}, \tag{5}$$

where m_{rad} is the mass radiated away as gravitational waves. This can be found using Einstein's energy equation.

$$m_{\text{rad}} = \frac{E}{c^2} \tag{6}$$

The radius of a black hole is dependent on its mass and is given by the Schwarzschild radius [3].

$$R_s = \frac{2GM}{c^2} \tag{7}$$

Using the Bekenstein-Hawking formula, the entropy of a black hole is given by [4]:

$$S = \frac{kA}{4l_p^2}, \tag{8}$$

where k is the Boltzmann constant, A is the surface area ($4\pi R_s^2$) and l_p is the Planck length ($\sqrt{\hbar G/c^3}$) [4]. Substituting these variables into the equation leads to an alternative version of the formula:

$$S = \frac{4\pi k G M^2}{c\hbar}, \tag{9}$$

where $\hbar$ is the reduced Planck constant.

Results

The masses of the supermassive black holes are $4.1 \times 10^6 M\odot$ and $1.7 \times 10^8 M\odot$ [1]. Using an initial separation of $9.46 \times 10^{15} m$ and $5.13 \times 10^{11} m$ as the final separation distance (sum of Schwarzschild radii of the individual black holes) [1], the energy emitted as gravitational waves during the inspiral is:

$$E = 1.8 \times 10^{53} J \tag{10}$$

Using equation 6, this equates to a mass loss of approximately $10^6 M\odot$. Substituting this value into equation 5 for m_{rad} gives the mass of the black hole formed after merger as:

$$M = 3.5 \times 10^{38} kg \tag{11}$$

Consequently the entropy of the black hole, found using equation 9 is:

$$S = 4.4 \times 10^{70} JK^{-1} \tag{12}$$

Similarly, the total entropy of the binary system before the inspiral is found by summing the entropy of the individual black holes. This is calculated as $4.2 \times 10^{70} JK^{-1}$. Therefore the increase in entropy of the system is approximately 4.8%.

Discussion

Analysing the results, the second law of thermodynamics is found to be upheld in the merger event, as the entropy increases. The actual change in entropy could be different as we have treated the supermassive black hole binary as an isolated system and have only considered energy loss in form of gravitational waves. In reality, angular momentum can be transferred to nearby stars during the inspiral [5]. Furthermore, gravitational waves have linear momentum and this could result in the black hole being "kicked" out of the evolving galaxy due to recoil. Such a phenomenon has been observed recently in the CID-42 system [6]. Future work could look into working out the "kick" velocity of the black hole after formation.

References

[1] Griffiths et al., *A3_3 Power Radiated from Merging Galaxies due to Gravitational Waves*, PST 15, (2016).

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