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A4_7 Who's up for a Ride on the Volcano?

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Abstract

Using Bernoulli's theorem for incompressible fluids we calculate the required pressure and velocity of gas and ash in a volcanic eruption chamber needed to propel Manny the mammoth skywards out of a lava-filled crater. We calculate that a pressure of 14.7 MPa, and an outflow velocity of 112.6 ms^{-1} are required.

Introduction

In the 2002 Blue Sky Studios' film "Ice Age", one of the main characters Manfred "Manny" (a woolly mammoth) plunges into a lava field when an ice bridge collapses above it. He is then catapulted back to safety by a convenient volcanic gas plume. The clip [1] doesn't explicitly show it, but we adopt a model of a high pressure fluid, flowing uniformly upwards from a gas chamber just below the surface through a volcanic vent.

We calculate the minimum pressure in the magma chamber that would be required to send Manny blasting off, considering the effects of confined fluid physics as well as air resistance.

Theory

We make the assumption that the outflow of the super-heated volcanic gas from the vent behaves like an incompressible fluid with associated density and pressure. The general equation for incompressible flow along a streamline is then given by Bernoulli's theorem:

$$B = \frac{P}{\rho} + \frac{v^2}{2} + gz. \qquad (1)$$

Here B is the constant Bernoulli integral, P, ρ and v are the fluid pressure, density and velocity respectively, g is gravitational acceleration (9.81 ms^{-2}) and z is the vertical difference between the ends of the streamline (depth below the vent). The internal and external flows in the gas are related by

$$\frac{P_{int}}{\rho_{gas}} + \frac{v_{int}^2}{2} - gz = \frac{P_{out}}{\rho_{gas}} + \frac{v_{out}^2}{2}. \qquad (2)$$

Given that the density ρ_{gas} of the lava doesn't change (on short time-scales), then the internal chamber pressure P_{int} is given in terms of the external pressure P_{out} (atmospheric, 101325 Pa) and velocity v_{out} by Equation 3. We take v_{int} to be negligible (since the volume of the chamber is so large).

$$P_{int} = P_{out} + \rho_{gas} \times \left(\frac{v_{out}^2}{2} + gz\right). \qquad (3)$$

To determine this value, we require the outflow velocity. We assume the volcanic plume erupts instantaneously at the point when Manny arrives, and that the collision is entirely elastic with the full force of the plume directed upwards.

Since Manny's fall direction is negative relative to the gaseous outflow, and he is propelled by the gas to approximately double the height of his starting position above the lava, the outflow velocity must be roughly three times his falling speed to give the required momentum change.

Considering air resistance, Manny would be falling with a velocity $v_{fall}(t)$ given by

$$v_{fall}(t) = \sqrt{\frac{2mg}{\rho_{air} C_d A}} tanh\left(t\sqrt{\frac{g\rho_{air} C_d A}{2m}}\right) \quad (4)$$

where m is Manny's mass (kg), ρ_{air} is the density of air (1.3 kgm^{-3}) [2], C_d is the drag coefficient, A is the cross sectional area (m^2), and t is the time (s).

Results

The clip [1] shows Manny off screen for approximately 7 seconds. We choose a time t of 4 s for his fall based on this clip, assuming a greater ascending velocity. To determine the magnitude of his falling velocity at the end of his descent with Equation 4, we use numerical values of: $m = 6000$ kg, the average mass of a male mammoth [3] (assuming ice mass is negligible); a drag coefficient of $C_d = 0.8$; and a cross sectional area of $A = 10$ m^2. (The rationale for these C_d and A values are shown in Discussion).

His velocity at contact with the volcanic plume v_{fall} is 37.5 ms^{-1}, giving an outflow velocity v_{out} of 112.6 ms^{-1} (three times $|v_{fall}|$). If we take an (average) gas density ρ_{gas} of 2300 kgm^{-3}[4] and vertical displacement z of 0 m we find an internal magma chamber pressure of 14.7 MPa.

Discussion

If this scenario existed outside the realms of cartoon animation, then it might be physically possible. Geysers and volcanic ash vents do erupt suddenly, and could conceivably have pressure of the order 15 GPa. However, it is highly unlikely that Manny could survive the immense g-forces, a result of the sudden direction change, yet he might survive the intense temperatures.

We have made the assumption that all of the momentum of the volcanic plume is transferred vertically to Manny atop his ice shelf, such that the area of the vent is less than the ice area, and there is no lateral momentum loss. If there were losses, then the rebound speed of Manny would be lower, possibly insufficient to survive. We also assume that the vent acts like a rocket nozzle so would create a plume with maximum velocity.

In Equation 3, every 10 m in z adds $\simeq 1.5\%$ (22 KPa) to the pressure, meaning the required pressure increases dramatically with depth. We consider a depth of 0 m to show the minimum pressure required.

The opposite is true of the values of C_d, the drag coefficient, and A the cross sectional area, used in Results. C_d could vary from around 0.5 to about 1.5 (depending on the size and shape of a mammoth, as well as how fluffy their coat is- this value is difficult to determine precisely). A could vary by $\pm$ 2 m^2 depending on the ice size. However, neither of these would alter the final velocity (Equation 4) by more than a factor of 1.5 so is not considered to be significant variation.

In another paper we would consider the effects of dispersal of volcanic gas kinetic energy from the vent, and investigate how quickly the ice he stands on is vaporised. This could be beneficial if it is rapid, as the mass loss would work similarly to the rocket-mass equation, and the (negligible) vapour pressure would work to boost the upwards velocity he achieves. Finally, we haven't considered gas turbulence nor sonic shock fronts that could occur at the vent velocities, so these would be avenues for further work.

References

[1] `https://youtu.be/RjJu1BA1Rv0` accessed on *10/11/16*.

[2] `https://goo.gl/2Akbzo` accessed on *13/11/16*.

[3] `https://goo.gl/MHt77b` accessed on *13/11/16*.

[4] Wilson et. al., (2011). "Volcanic ash impacts on critical infrastructure". Physics and Chemistry of the Earth. (Page 8).