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A2_4 Playing Hungry Hungry Hippos with Black Holes

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Abstract

We investigate the relationship between the mass and density of a black hole. It leads to the conclusion that a black hole of mass 2.7×10^{38} kg has the same density as water. By taking this further we find that for supermassive black holes of mass larger than 2.3×10^{38} kg a star similar to the Sun in terms of mass and radius is no longer tidally disrupted before reaching the black hole and is instead swallowed whole.

Introduction

We discuss the properties of a Schwarzschild black hole, a black hole with no charge or angular momentum. For such a black hole the volume is dictated by the Schwarzschild radius, the point at which the escape velocity from the gravity of the black hole is greater than the speed of light. In this paper we will show how a black hole can have a density equal to that of water and can eventually have a density so low that it swallows stars whole before it is capable of tidal disruption.

Theory

The Schwarzschild radius is the point of no return for an object near a black hole, which is expressed in Eq (1)

$$R_{sch} = \frac{2MG}{c^2} \tag{1}$$

Where M is the mass of the black hole, G is the gravitational constant and c is the speed of light [1].

For a star orbiting a black hole, where the black hole is either accreting matter and growing in size, or the star is spiralling inwards towards the black hole, in most situations it will be torn apart due to tidal disruption long before it reaches the Schwarzschild radius. The tidal radius describes the distance (from the black hole) for a mass (e.g. a star) at which the total centrifugal and gravitational force is stronger than the stars self-binding gravitational force. This is what tears the star apart [2]. The tidal radius R_T is given by Eq (2)

$$R_T = R_* \left(\frac{M}{M_*}\right)^{(1/3)} \tag{2}$$

Where M_* is the mass of the star and R_* is the radius of the star [3]. Assuming that the black hole is a sphere with uniform density, where

$$\rho = \frac{M}{V} = \frac{3M}{4\pi R_{sch}^3}, \tag{3}$$

we can evaluate Eq's (1), (2) and (3) to find the relationship between M , R_T, R_{sch} and the density ρ inside a black hole. If we rearrange Eq (1) for M and substitute it into Eq (3) we get the following proportionality

$$\rho \propto R_{sch}^{-2} \tag{4}$$

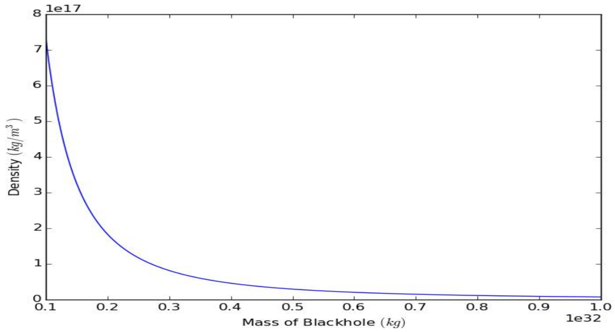

Figure 1: The relationship between density and mass inside a black hole.

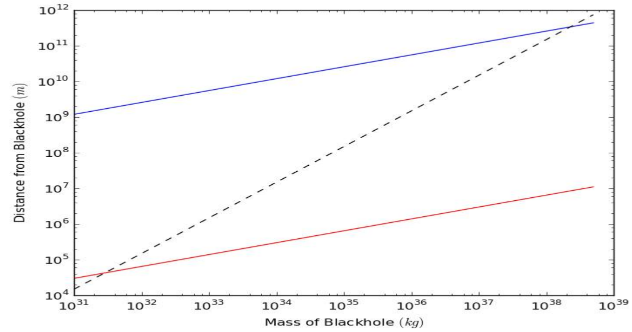

Figure 2: The black hole masses at which $R_T = R_{sch}$ for the Sun (blue line) and a neutron star (red line), dashed line is the R_{sch}

This means that the density of the black hole decreases as its volume increases, rather than remaining constant. This is because $V \propto R_{sch}^3$. Fig (1) depicts this asymptotic density evolution. Following up from Eq (1) we find that the most massive black holes are actually the least dense. This leads us to the unusual conclusion that a black hole with a mass of 2.7×10^{38} kg has the density of water (from Eq's (1) and (3)).

By rearranging Eq (2) for M and subbing in for ρ we find that

$$\rho \propto R_T^3. \tag{5}$$

So if the tidal radius decreases with decreasing density and the Schwarzschild radius increases with decreasing density, at a certain mass we find that the Schwarzschild radius is no longer inside the tidal radius for a given star. For a large enough black hole our star can orbit straight into the black hole without experiencing any noticeable tidal forces. As such, we are interested in the various masses of stars and the associated black hole mass at which the inequality in Eq (6) is true. So, by definition, the point at which a star is swallowed instead of tidally disrupted is

$$R_{sch} > R_T. \tag{6}$$

The point at which this occurs is governed by Eq (1) and (2). By setting $R_{sch} = R_T$ and rearranging for M. Fig (2) shows that for a black hole to swallow our Sun it would require a mass larger than $M = 2.3 \times 10^{38}$ kg, and for a black hole to swallow a typical neutron star with radius 20 km and mass $1.5M_{Sun}$ it would require a mass larger than $M = 2.9 \times 10^{31}$ kg [4]. Even though the mass of the neutron star is more than our Sun, a black hole will swallow it before it will swallow the Sun.

Conclusion

We have found that whilst a black hole is formed from very massive and dense objects, the growth of a black hole actually reduces the density. We have shown that it is even possible for a common star to not experience tidal disruption when approaching the supermassive black hole, and is instead swallowed whole.

References

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/astro/blkhol.html accessed on 17/10/16

[2] http://www.seramarkoff.com/2016/02/why-tidal-disruptions-are-interesting/ accessed on 17/10/16

[3] https://ned.ipac.caltech.edu/level5/March01/Battaner/node13.html accessed on 17/10/16

[4] http://science.nationalgeographic.com/science/space/solar-system/neutron-stars/ accessed on 17/10/16