Journal of Physics Special Topics
An undergraduate physics journal

P3_5 On the minimum mass for a Death Star's power system

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November 14, 2016

Abstract

We examine the minimum mass necessary to build the power system for a *Death Star*, as shown in the *Star Wars* series of films, using current technology. Considering its powerplant as a Carnot engine we are able to find a balance between the necessary mass of reactor and heat radiators so as to reduce the overall mass of the Death Star's power system to 6.8×10^{25} kg, around 930 lunar masses to 2 significant figures.

Introduction

The Star Wars [1] films depict a vessel, called the Death Star, capable of destroying planets with a laser. This laser has a power 5.7×10^{26} W, calculated from the beam's energy [2] and the 3.5 s of screen-time for which the beam is in operation [1]. We will assume that the Death Star's power system consists of 3 main components: a nuclear fission reactor, outputting thermal power P_n at temperature T_h, with mass M_n; a heat radiator of area A, mass M_r, and temperature T_c being used to dump waste thermal power P_w into space; and a laser consuming electrical power P_l, to produce the laser beam. High power lasers commonly have an efficiency of around 30% [3]. We therefore assume that for a 5.7×10^{26} W beam the laser consumes $P_l = 1.9 \times 10^{27}$ W. Power wasted within the laser is disposed of via systems built into the laser. As this paper only calculates the mass of the Death Star's power systems, the design and mass of the laser and its built-in systems for removing the waste heat generated within the laser are ignored. We assume that the Death Star operates in a steady state; not needing time to cool or recharge between shots.

Carnot engines and radiators

A Carnot engine gives the most efficient conversion of a thermal gradient into useful power. The efficiency, ϵ_c, for such an engine operating between a hot reservoir and a cold reservoir is given by Eq.(1) [4].

$$\epsilon_c = 1 - \frac{T_c}{T_h} = \frac{P_l}{P_n} \tag{1}$$

On our Death Star the hot reservoir is the reactor, the cold reservoir is the radiator emitting heat into space and the useful power is supplied to the laser. Eq.(2) follows from considering the conservation of energy.

$$P_w = P_n - P_l \tag{2}$$

As the radiator emits power P_w and by assuming an emissivity of 1, we get Eq.(3) [4], with A the area of the radiator, and σ the Stefan-Boltzmann constant.

$$P_w = A\sigma T_c^4 \tag{3}$$

We also define ϕ as the area per unit mass of the radiator, and θ as the power output per unit mass of the reactor, hence Eqs.(4) and (5).

$$A = M_r\phi \quad (4) \qquad P_n = M_n\theta \quad (5)$$

Then we set Eq.(3) equal to Eq.(2), rearrange for T_c, and substitute the result into Eq.(1). By replacing A and P_n with Eq.(4) and Eq.(5) respectively, we get Eq.(6).

$$\frac{P_l}{M_n\theta} = 1 - \frac{\left(\frac{M_n\theta - P_l}{M_r\sigma\phi}\right)^{\frac{1}{4}}}{T_h} \qquad (6)$$

The total mass of the power system M_t is given by $M_t = M_n + M_r$. Rearranging Eq.(6) for M_r and substituting this into M_t gives Eq.(7).

$$M_t = M_n + \frac{M_n\theta - P_l}{\sigma\phi T_h^4\left(1 - \frac{P_l}{M_n\theta}\right)^4} \qquad (7)$$

Minimising mass

To find the minimum total mass it would appear logical to find the differential $\frac{\partial M_t}{\partial M_n}$ and set it equal to 0 to find a turning point. Unfortunately, the differential in question is a fifth order polynomial and cannot be solved analytically. We therefore adopt a graphical method to find the turning point. Before the graph can be plotted values for the variables θ, ϕ, P_l and T_h must be fixed. P_l has been calculated in the introduction.

A typical nuclear reactor core has $\theta = 40$ kWkg^{-1}; this value was calculated by dividing the thermal power output of a reactor core by the core's fuel mass [5]. For current fission reactors T_h is around 600 K, as found from the outlet temperature [5].

We use $\phi = 0.036$ m^{2}kg^{-1}. This was derived from the area and mass of the thermal control radiators on the ISS [6] which are 1100 kg with a double sided area of 23 m x 3.4 m.

Fig.(1) shows Eq.(7) for reactor masses ranging from 1×10^{23} kg to 5×10^{23} kg. The turning point is marked and found to occur at $M_n = 1.9 \times 10^{23}$ kg, at which point $M_t = 6.8 \times 10^{25}$ kg. The difference of these masses is the value of M_r required for the radiators which act as the cold reservoir.

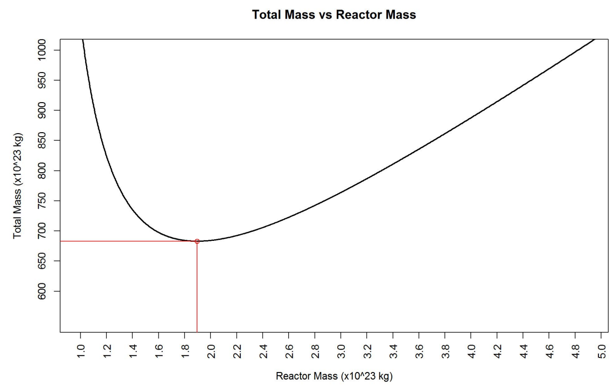

Figure 1: A plot of the relationship between the total mass, M_t, and mass of the reactor, M_n. The minimum total mass occurs at the turning point, marked by the red circle and lines.

Conclusion

We find that the minimum mass for a Death Star's power system built using current technology is $M_t = 6.8 \times 10^{25}$ kg. It must be noted that a complete Death Star would also include the mass of: the laser, the laser's built-in cooling equipment, a structural framework and a non-zero-mass Carnot engine. A power source of higher θ or T_h than current technology could reduce the mass needed. We conclude that, on the grounds of the mass required, a complete Death Star using current technology would not be easy to confuse with a "small moon" [1].

References

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